Fluid Statics

Lecture - 2

Fluid Statics

- Fluid Statics means fluid at rest.
- At rest, there are no shear stresses, the only force is the normal force due to pressure is present.
- **Pressure** is defined as:
- "Force per Unit Area" Or

  "The amount of force exerted on a unit area of a substance or on a surface."
- This can be stated by the equation:

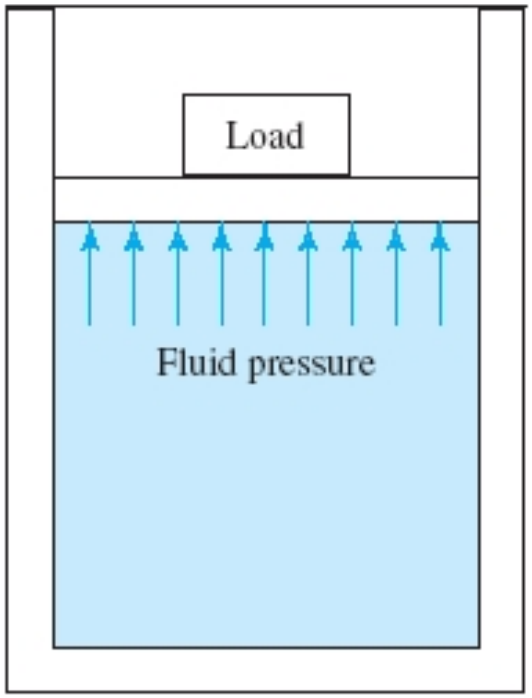
$$p = \frac{F}{A} \text{ (For Finite Area)} \qquad p = \frac{dF}{dA} \text{ (For Infinite Area)}$$

- Units : N/m^2(Pa), lbs/ft^2 (psf), lbs/in^2 (psi)

Example

A load of 200 pounds (lb) is exerted on a piston confining oil in a circular cylinder with an inside diameter of 2.50 inches (in). Compute the pressure in the oil at the piston.

Solution:



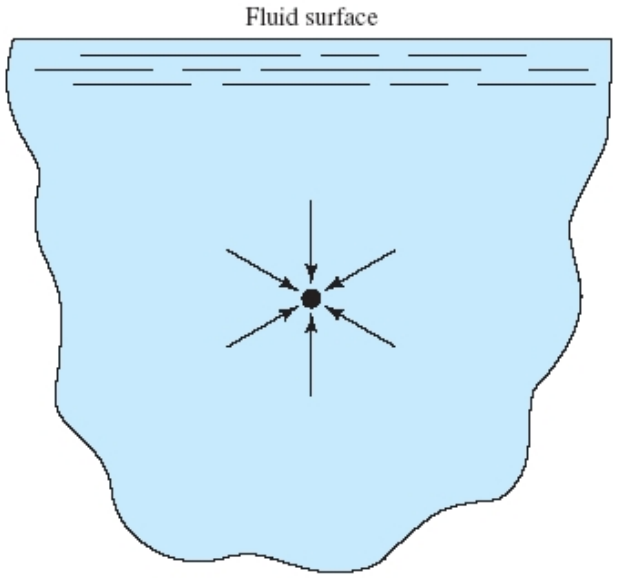
$$A = \pi D^2/4 = \pi(2.50\ \text{in})^2/4 = 4.91\ \text{in}^2$$

$$p = \frac{F}{A} = \frac{200\ \text{lb}}{4.91\ \text{in}^2} = 40.7\ \text{lb/in}^2$$

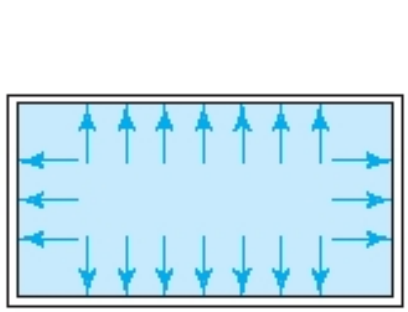
Principles about Pressure

- Two important principles about pressure were described by **Blaise Pascal**, a seventeenth-century scientist:

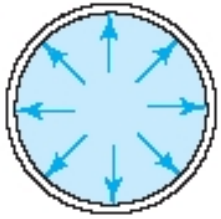
1. Pressure acts uniformly in all directions on a small volume of a fluid.
2. In a fluid confined by solid boundaries, pressure acts perpendicular to the boundary.



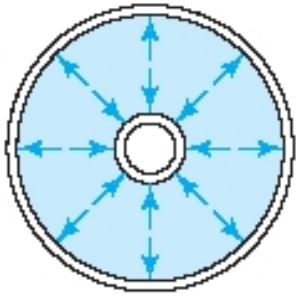
Direction of fluid pressure on boundaries



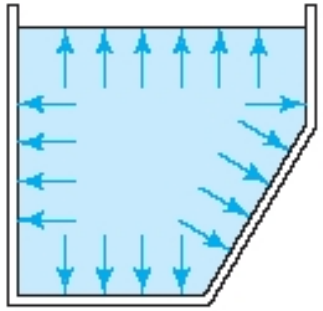
(a) Furnace duct



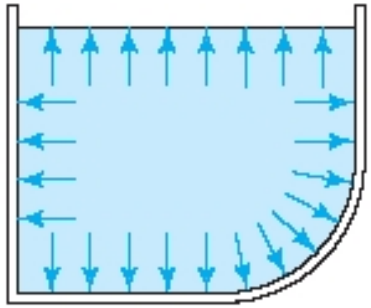
(b) Pipe or tube



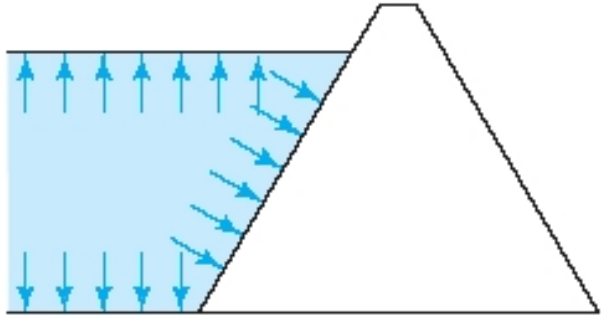
(c) Heat exchanger (a pipe inside another pipe)



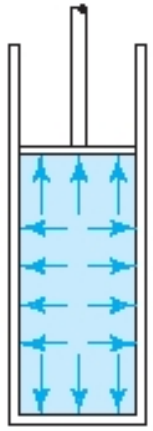
(d) Reservoir



(e) Swimming pool



(f) Dam



(g) Fluid power cylinder

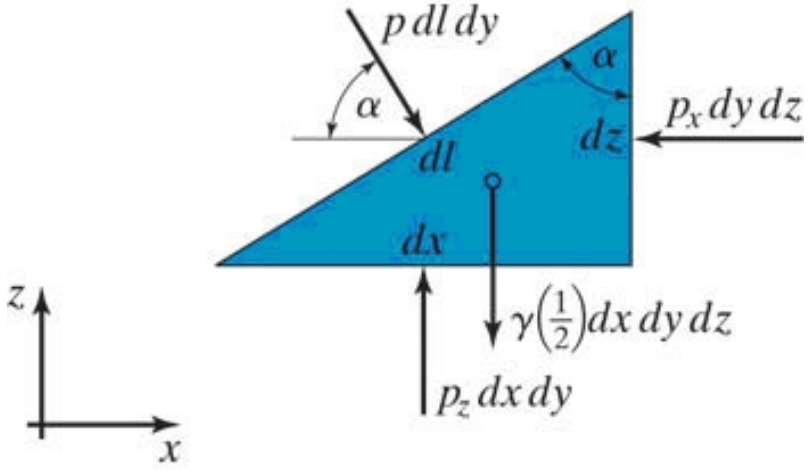
Pressure at a Point is same in All direction (Pascal's Law)

1. Consider a wedge shaped element of fluid at rest.
2. Thickness = dy (perpendicular to plane of paper).
3. Let p = Avg. pressure in any direction.
4. p_x and p_z are avg. pressure in horizontal and vertical direction.
5. Forces in y direction need not be considered because they cancel.
6. No tangential force is involved, since the fluid is at rest.
7. Equilibrium Condition : Sum of the force components on element in any direction must be equal to Zero.
8. $\sum F_x = 0; \quad p \cos\alpha \, dl \, dy - p_x \, dy \, dz = 0$

Since $dz = dl \cos\alpha$

Therefore;

$p = p_x$



Pressure at a Point is same in All direction

9. $\sum F_z = 0;\ p_z\ dx\ dy\ - p\ dl\ dy \sin\alpha - \frac{1}{2}\gamma\ dx\ dy\ dz = 0$

Neglecting 3rd term due to higher order.

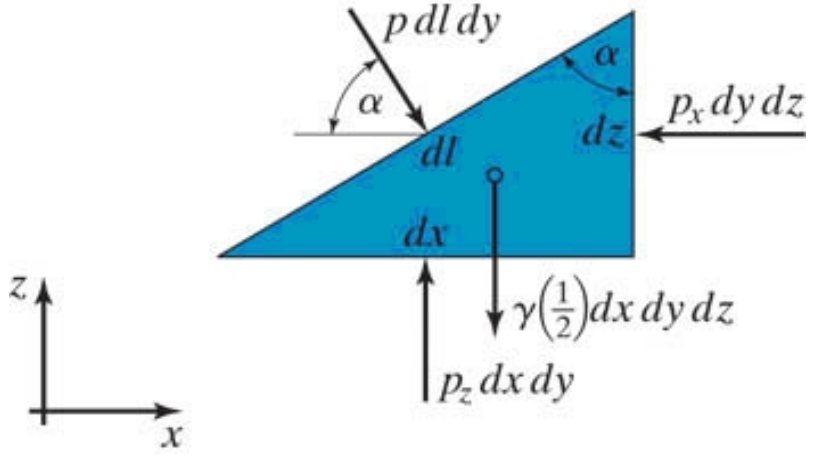
Since $dx = dl \sin\alpha$

Therefore;

$p = p_z$

10. We can also proof p = py by considering a three dimensional case.

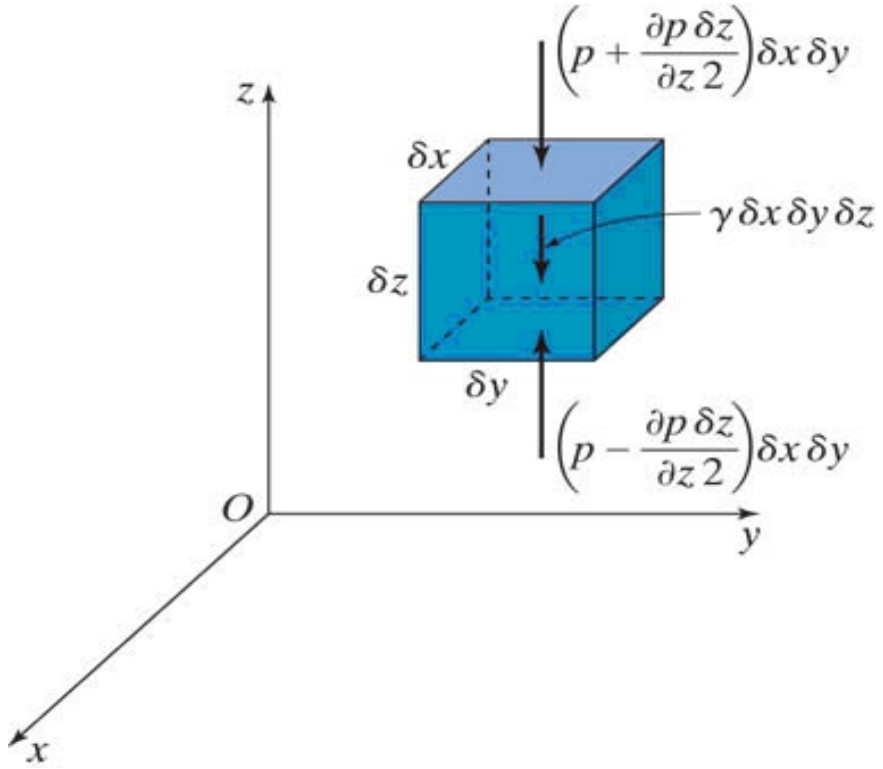
Thus pressure at any point in a fluid at rest is same in all direction.



Variation of Pressure in a Static Fluid (Hydrostatic Law)

- It states:

  "Rate of increase of pressure in vertical downward direction must be equal to Specific Weight of fluid at that point."



Variation of Pressure in a Static Fluid (Hydrostatic Law)

1. Consider an element of static fluid.
2. Assume density of fluid to be constant within the element (Since the element is very small).
3. Pressure at the center of element is p.
4. Dimensions of element are δx δy and δz.
5. Force acting in vertical direction are :
    a. Body force : the action of gravity on the mass within the fluid.
    b. Surface force : transmitted from the surrounding fluid.
6. If the forces are summed in the horizontal direction, that is x and y, the only forces acting are the pressure forces on the vertical faces of element.

7. To satisfy $\sum F_x = 0$ and $\sum F_y = 0$, the presssures on the opposite vertical faces must be equal.

8. Thus $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$

9. Summing forces in the vertical direction and putting it equal to zero.

$$\sum F_z = \left(p - \frac{\partial p}{\partial z}\frac{\delta z}{2} \right)\delta x \delta y - \left(p + \frac{\partial p}{\partial z}\frac{\delta z}{2} \right)\delta x \delta y - \gamma \delta x \delta y \delta z = 0$$

After simplification :

$$\frac{\partial p}{\partial z} = -\gamma$$

Since p is independent of x and y, we can write above equation as :

$$\frac{dp}{dz} = -\gamma$$

This is the general expression that relates variation of pressure in a static fluid to vertical position. The minus sign indicates that as z gets larger (increasing elevation), the pressure gets smaller.

To evaluate the pressure anywhere in a fluid at rest, we must intergate previous equation between appropriate chosen limits.

$$dp = \gamma\, dz$$

$$\int dp = \int \gamma\, dz$$

$$p = \gamma\, z$$

Say; $z = h$

$$p = \gamma\, h$$

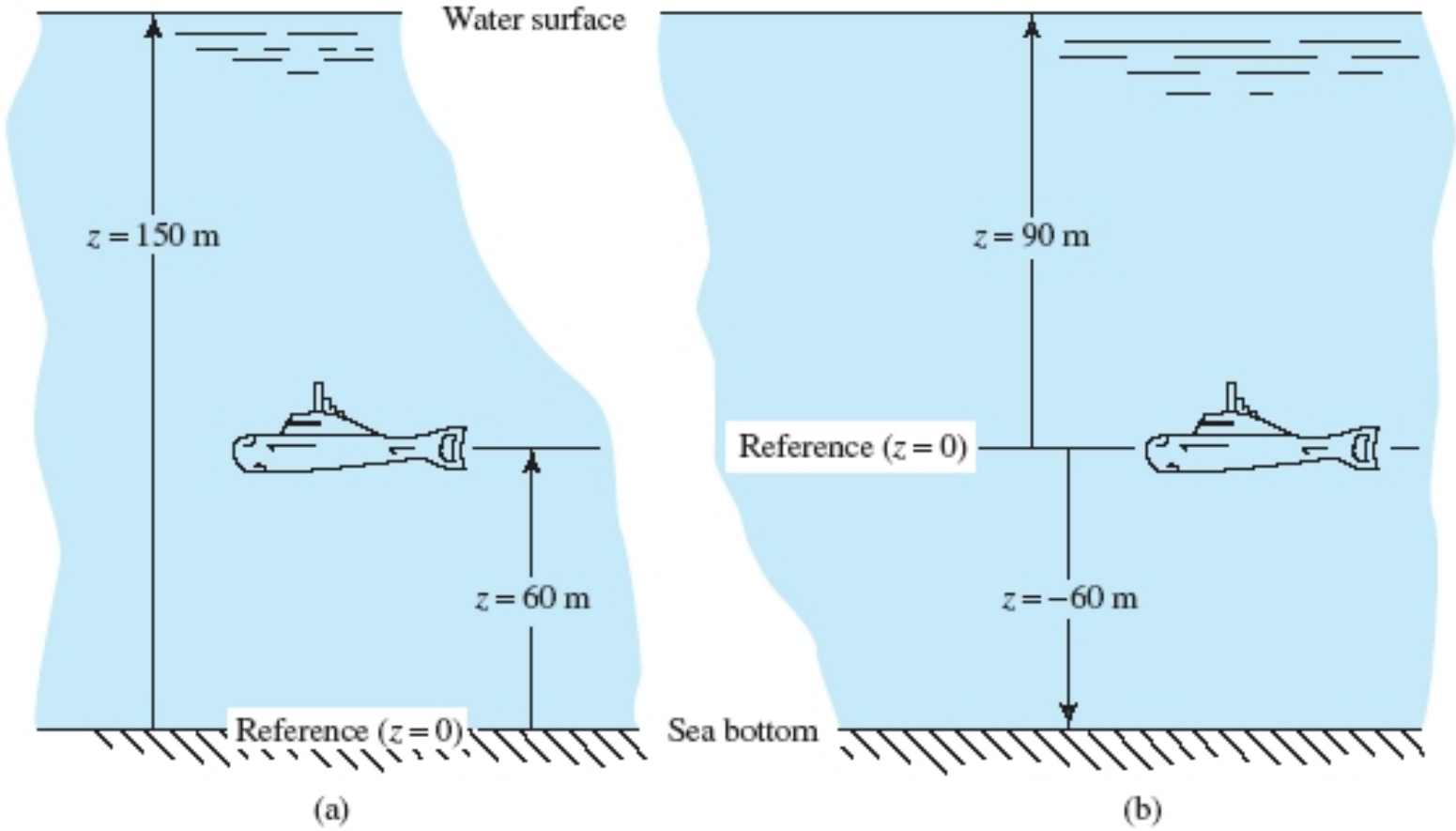
Or

$$h = \frac{p}{\gamma} \quad \text{(Pressure Head)}$$

Pressure expressed in Height of Fluid

- The term *elevation* means the vertical distance from some reference level to a point of interest and is called z.
- A *change* in elevation between two points is called h. Elevation will always be measured positively in the upward direction.
- In other words, a higher point has a larger elevation than a lower point.
- Fig shows the illustration of reference level for elevation.

Relationship between Pressure and Elevation:

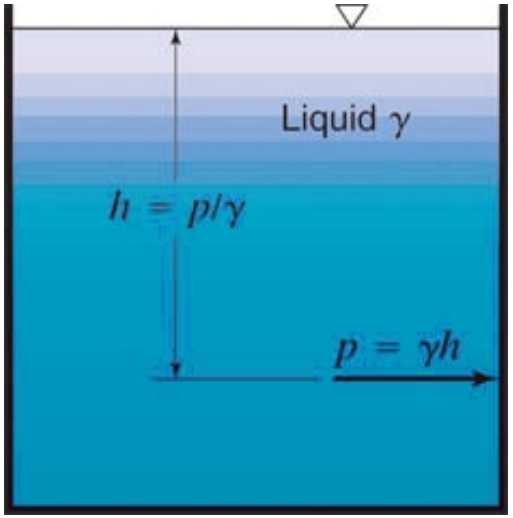


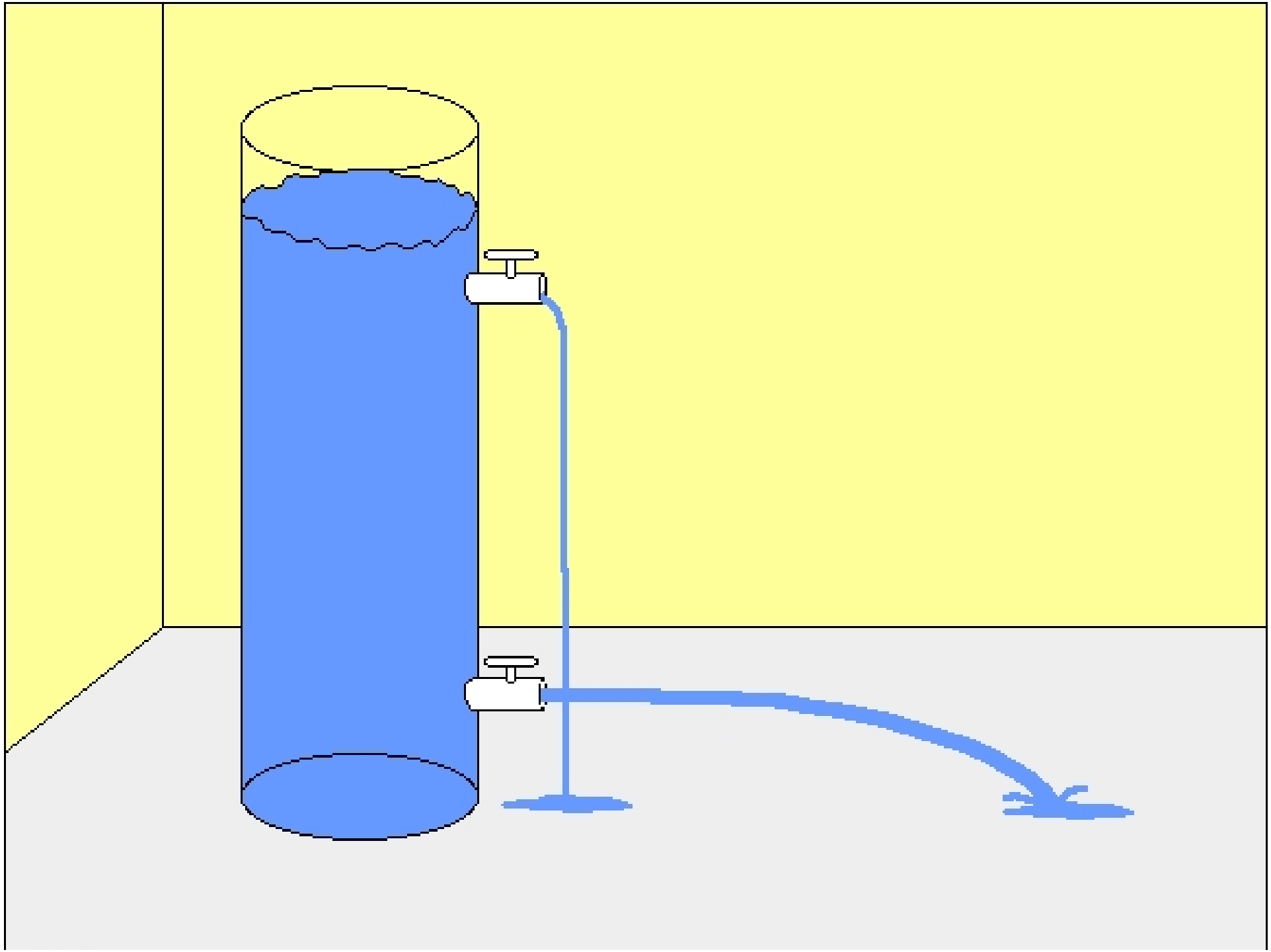
Pressure Head

- It is the pressure expressed in terms of height of fluid.

$$h = \frac{p}{\gamma}$$

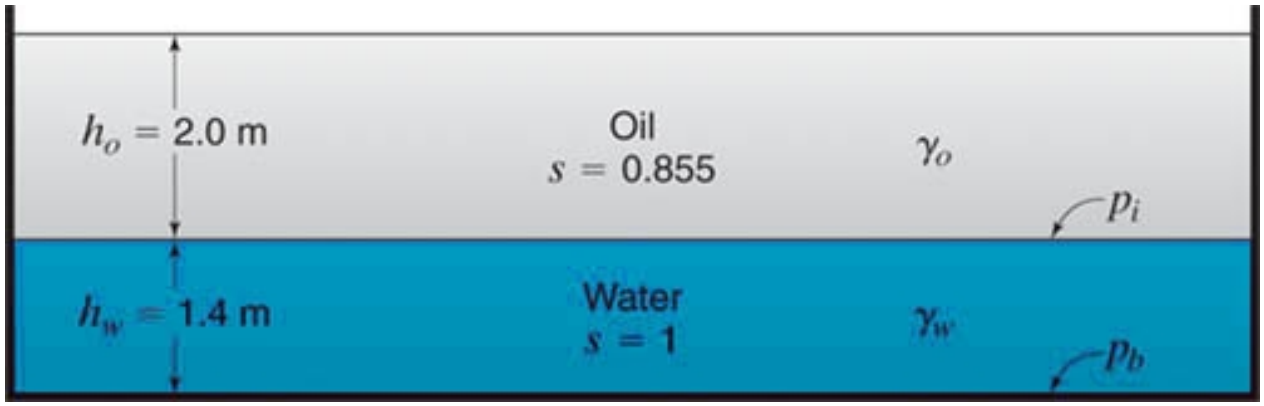
- h=p/γ represents the energy per unit wt. stored in the fluid by virtue of pressure under which the fluid exists. This is also called the elevation head or potential head.





Example

An open tank contains water 1.40m deep covered by a 2m thick layer of oil (s=0.855). What is the pressure head at the bottom of the tank, in terms of a water column?



Solution:

$\gamma_w = 9.81 kN/m^3$

$\gamma_o = 0.855x9.81 = 8.39 kN/m^3$

For interface : $\mathrm{p_i} = \gamma_o h_o = 8.39x2 = 16.78 kN/m^2$

for water equivalent of oil :

$$\mathrm{h_{oe}} = \frac{p_i}{\gamma_w} = \frac{16.78}{9.81} = 1.710m$$

So $\mathrm{h_{we}} = h_w + \mathrm{h_{oe}} = 1.40 + 1.710 = 3.11m$

SOLUTION– 2

$$p_b = \gamma_o h_o + \gamma_w h_w$$

$$= (8.39)2 + 9.81(1.4)$$

$$= 30.51 \mathrm{kN/m^2}$$

$$h_{we} = \frac{p_b}{\gamma_w} = \frac{30.51}{9.81} = 3.11m$$

Exercises (Assignment):

1. An open tank contains 5 m water covered with 2 m of oil (γ=8kN/m3). Find the gauge pressure (a) at interface between the liquids and at bottom of the tank.
2. An open tank contains 7ft of water covered with 2.2ft oil (s=0.88). Find the gauge pressure (a) at the interface between the liquids and (b) at the bottom of the tank.
3. If air had a constant specific weight of 12N/m3and were incompressible, what would be the height of air surrounding the earth to produce a pressure at the surface of 101.3 kPa abs?

Absolute and Gage Pressure

- **Atmospheric Pressure:** It is the force per unit area exerted by the weight of air above that surface in the atmosphere of Earth (or that of another planet). It is also called as barometric pressure.
- **Gauge Pressure:** It is the pressure, measured with the help of pressure measuring instrument in which the atmospheric pressure is taken as Datum (reference from which measurements are made).
- **Absolute Pressure:** It is the pressure equal to the sum of atmospheric and gauge pressures. Or
- If we measure pressure relative to absolute zero (perfect Vacuum) we call it absolute pressure.
- Vacuum: If the pressure is below the atmospheric pressure we call it as vacuum.

$$p_{abs} = p_{atm} + p_{gage}$$

Measurement of Pressure

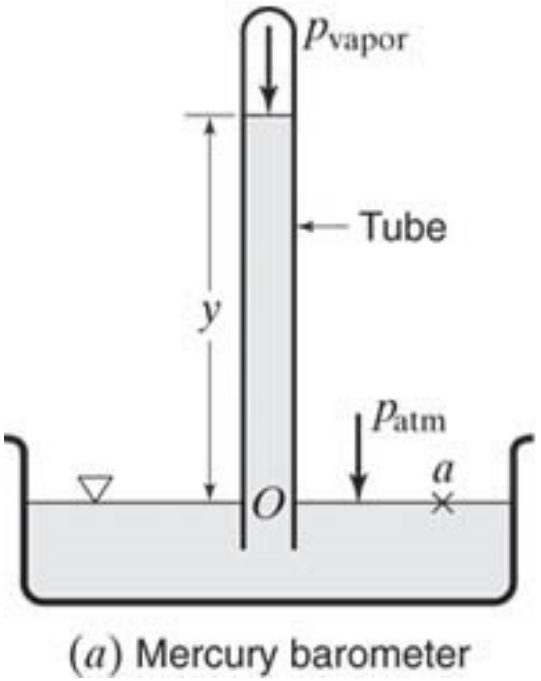
- There are many ways to measure pressure in a fluid. Some are discussed here:

1. Barometers
2. Bourdon gauge
3. Pressure transducers
4. Piezometer Column
5. Simple Manometers
6. Differential Manometers

1. Barometers:

- To measure the atmospheric pressure.
- Procedure:

1. Immerse the open end of tube in a liquid which is open to atmosphere.
2. The liquid will rise in the tube if we exhaust air from the tube.
3. If all the air is removed and the tube is long enough, than only pressure on the surface is the vapour pressure and liquid will reach its max. possible height (y).



(*a*) Mercury barometer

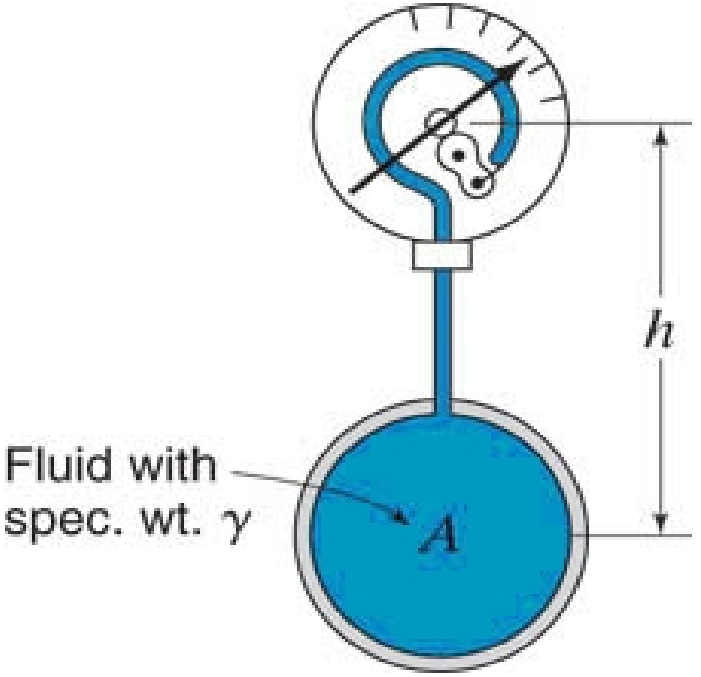
$$p_O = p_a = p_{atm} = \gamma y + p_{vapour}$$

If the vapour pressure on liquid surface in tube is negligible than :

$$p_{atm} = \gamma y$$

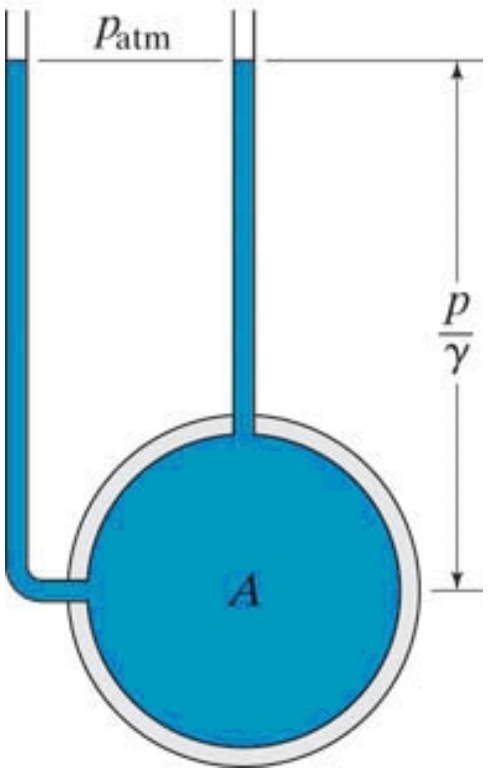
2. Bourdon Gauge:

- The pressure, above or below the atmospheric pressure, may be easily measured with the help of a bourdon's tube pressure gauge.
- It consists on an elliptical tube: bent into an arc of a circle. This bent up tube is called Bourdon's tube.
- Tube changes its curvature with change in pressure inside the tube. Higher pressure tends to "straighten" it.
- The moving end of tube rotates needle on a dial through a linkage system.



3. Piezometer Column/Tube:

- A piezometer tube is the simplest form of instrument, used for measuring, moderate pressure.
- It consists of long tube in which the liquid can freely rise without overflowing.
- The height of the liquid in the tube will give the pressure head (p/γ) directly.
- To reduce capillary error, the tube error should be at least 0.5 in.



4. Manometer:

- Manometer is an improved form of a piezometer tube. With its help we can measure comparatively high pressures and negative pressure also. Following are few types of manometers.

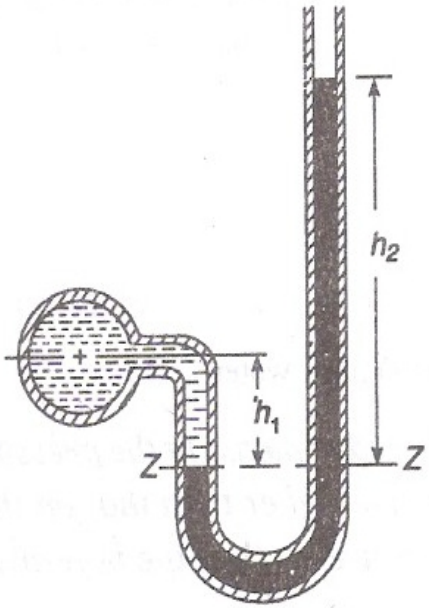
1. Simple Manometer
2. Micro-manometer
3. Differential manometer
4. Inverted differential manometer

Simple Manometer:

- It consists of a tube bent in U-Shape, one end of which is attached to the gauge point and the other is open to the atmosphere.
- Mercury is used in the bent tube which is 13.6 times heavier than water. Therefore it is suitable for measuring high pressure as well.

Procedure:

1. Consider a simple Manometer connected to a pipe containing a light liquid under high pressure. The high pressure in the pipe will force the mercury in the left limb of U-tube to move downward, corresponding the rise of mercury in the right limb.



(a) Positive pressure

Simple Manometer:

2. The horizontal surface, at which the heavy and light liquid meet in the left limb, is known as datum line.

   Let h1 = height of light liquid in the left limb above datum.

   h2 = height of heavy liquid in the right limb above datum.

   h= Pressure in the pipe, expressed in terms of head of water.

   s1=Sp. Gravity of light liquid.

   s2=Sp. Gravity of heavy liquid.

3. Pressure in left limb above datum = h +s1h1
4. Pressure in right limb above datum = s2h2
5. Since the pressure is both limbs is equal So,

   h +s1h1 = s2h2

   h= (s2h2 - s1h1)

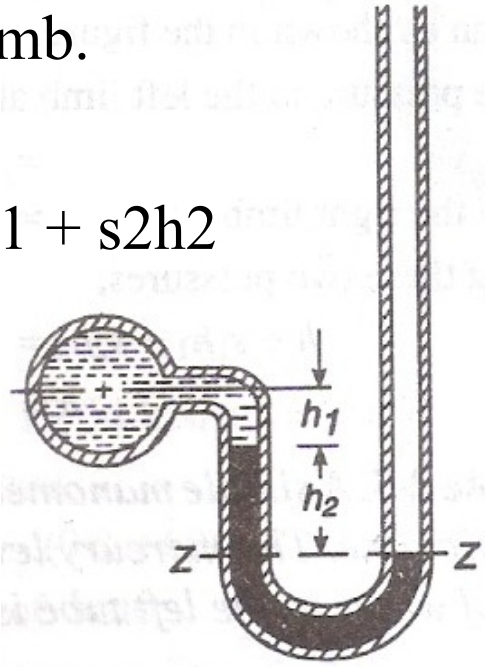
Simple Manometer:

To measure negative pressure:

In this case negative pressure will suck the light liquid which will pull up the mercury in the left limb of U-tube. Correspondingly fall of liquid in the right limb.

6. Pressure in left limb above datum = h +s1h1 + s2h2
7. Pressure in right limb = 0
8. Equating, we get

   h = -s1h1-s2h2 = -(s1h1+s2h2)

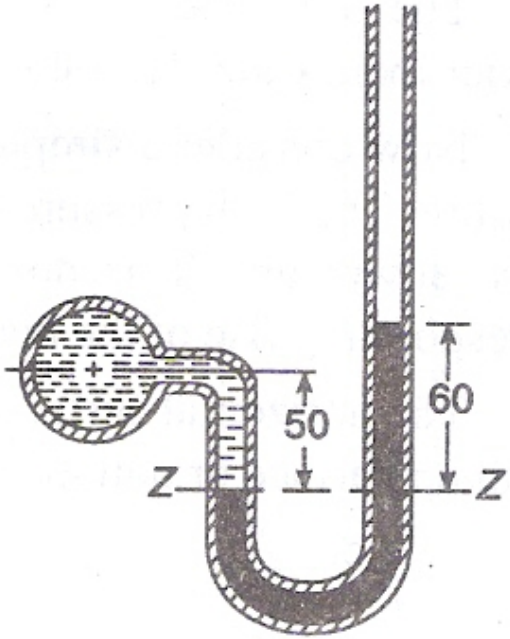


(b) Negative pressure

Example

A simple manometer containing mercury is used to measure the pressure of water flowing in a pipeline. The mercury level in the open tube is 60mm higher than that on the left tube. If the height of water in the left tube is 50mm, determine the pressure in the pipe in terms of head of water.

Solution:



Pressure head in the left limb above Z - Z

$$= \mathrm{h} + \mathrm{s}_1 h_1 = h + (1x50)$$

$$= \mathrm{h} + 50 \text{ mm}$$

Pressure head in the right limb above Z - Z

$$= s_2 h_2 = 13.6x60$$

$$= 816 \text{ mm}$$

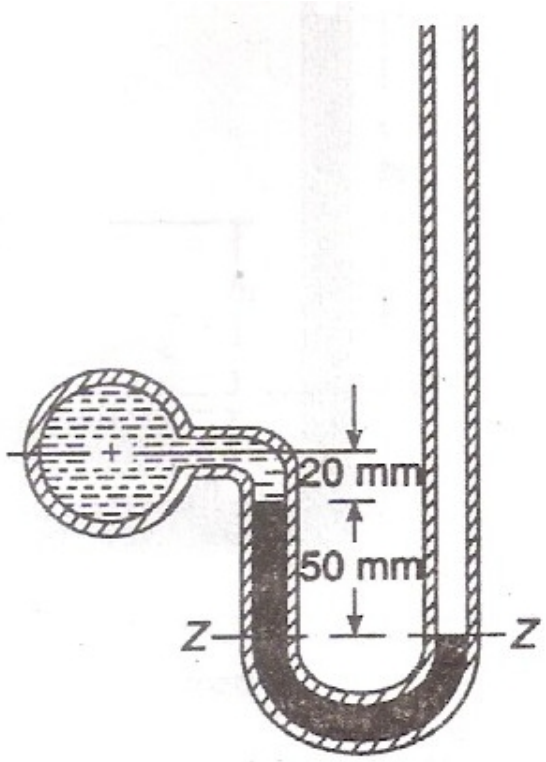
Equating;

$$\mathrm{h} + 50 = 816$$

$$\mathrm{h} = 766 \text{ mm}$$

Example

A simple manometer containing mercury was used to find the negative pressure in pipe containing water. The right limb of the manometer was open to atmosphere. Find the negative pressure, below the atmosphere in the pipe.



Solution:

Pressure head in the left limb above Z - Z

$$= h + s_1 h_1 + s_2 h_2 = h + (1x50) + (13.6x50)$$

$$= h + 700 \text{ mm}$$

Pressure head in the right limb above Z - Z

$$= 0$$

Equating;

$$h + 700 = 0$$

$$h = -700 \text{ mm} = -7\text{m}$$

Gauge pressure in the pipe $= p = \gamma h$

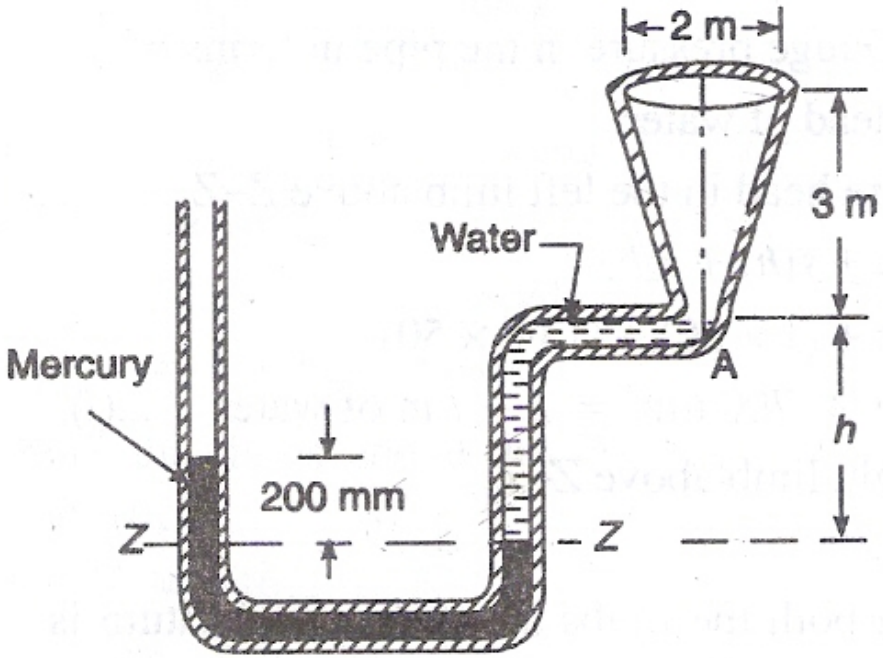
$$9.81x(-7) = -68.67\text{kN/m}^2$$

$$= -68.67\text{kPa}$$

$$= 68.67\text{kPa (Vacuum)}$$

Example

Figure shows a conical vessel having its outlet at A to which U tube manometer is connected. The reading of the manometer given in figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.



Solution:

$h_2 = 200mm = 0.2m$

$s_1 = 1$ and $s_2 = 13.6$

Let h = Pressure head of mercury in terms on head of water.

1. Let us consider the vessel is to be empty and Z - Z be the datum line.

Pressure head in the right limb above Z - Z

$= s_1 h_1 = 1xh = h$

Pressure head in the left limb above Z - Z

$= s_2 h_2 = 13.6x0.2 = 2.72m$

Equating; $h = 2.72m$

2. Consider the vessel to be completely filled with water.

As a result, let the mercury level goes down by x meters in the right limb, and the mercury level go up by the same amount in the left limb.

Therefore total height of water in the right limb

$= x + h + 3 = x + 2.72 + 3 = x + 5.72$

Pressure head in the right limb $= 1(x + 5.72) = x + 5.72$

We know that manometer reading in this case :

$= 0.2 + 2x$

Pressure head in the left limb

$= 13.6\,(0.2 + 2x) = 2.72 + 27.2x$

Equating the pressures :

$x + 5.72 = 2.72 + 27.2x$

$x = 0.115\text{m}$

and manometer reading $= 0.2 + (2x0.115) = 0.43\text{m} = 430\text{ mm}$

Differential Manometer:

- It is a device used for measuring the difference of pressures, between the two points in a pipe, on in two different pipes.
- It consists of U-tube containing a heavy liquid (mercury) whose ends are connected to the points, for which the pressure is to be found out.

Procedure:

- Let us take the horizontal surface Z-Z, at which heavy liquid and light liquid meet in the left limb, as datum line.
- Let, h=Difference of levels (also known as differential manomter reading)

  ha, hb= Pressure head in pipe A and B, respectively.

  s1, s2= Sp. Gravity of light and heavy liquid respectively.

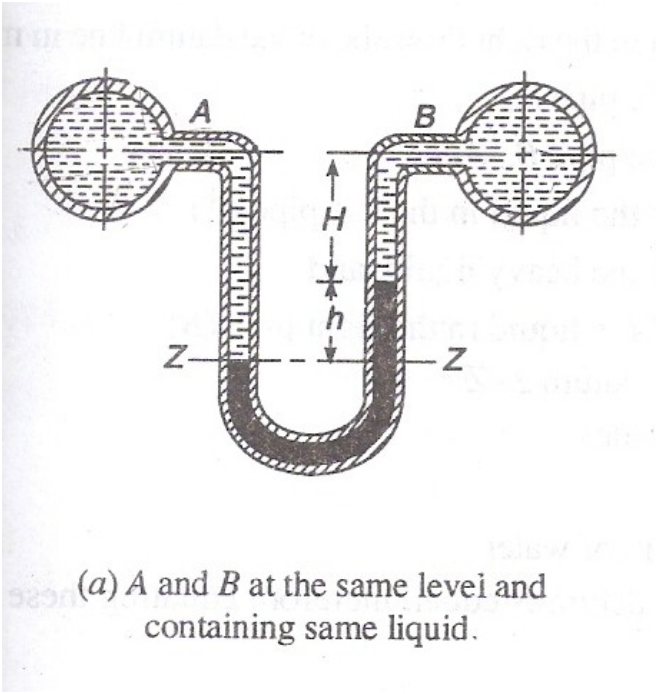
Differential Manometer:

1. Consider figure (a):
2. Pressure head in the left limb above
   Z-Z = ha+s1(H+h)= ha+s1H+s1h

3. Pressure head in the right limb above
   Z-Z = hb+s1H+s2h
4. Equating we get,

   ha+s1H+s1h = hb+s1H+s2h

   ha-hb=s2h-s1h = h(s2-s1)



(*a*) *A* and *B* at the same level and containing same liquid.

Differential Manometer:

Two pipes at different levels:

1. Pressure head in the left limb above Z-Z = ha+s1h1
2. Pressure head in the right limb above Z-Z = s2h2+s3h3+hb
3. Equating we get,

   ha+s1h1 = s2h2+s3h3+hb

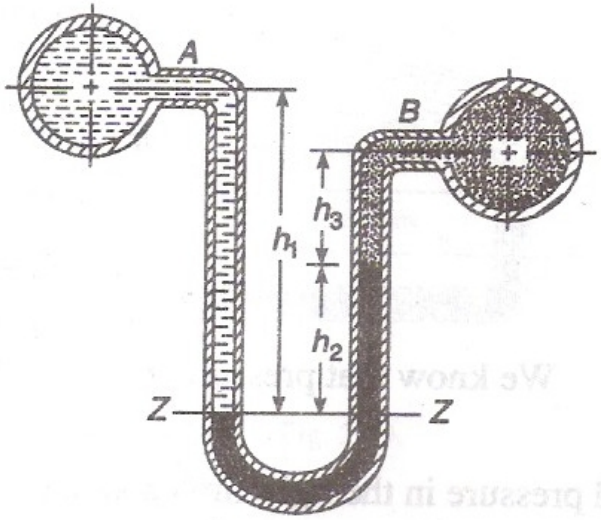
Where;

h1= Height of liquid in left limb

h2= Difference of levels of the heavy liquid in the right and left limb (reading of differential manometer).

h3= Height of liquid in right limb

s1,s2,s3 = Sp. Gravity of left pipe liquid, heavy liquid, right pipe liquid, respectively.



(*b*) *A* and *B* at different levels and containing different liquids.

Example

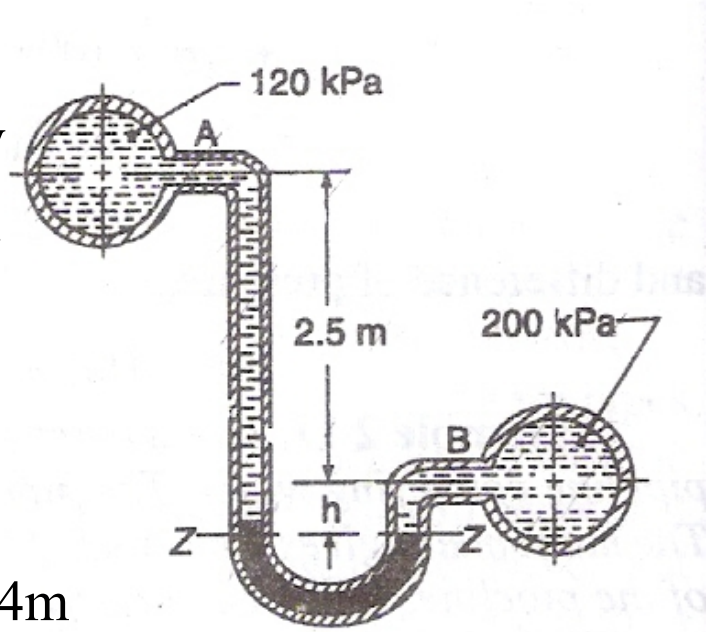
A U-tube differential manometer connects two pressure pipes A and B. The pipe A contains carbon Tetrachloride having a Sp. Gravity 1.6 under a pressure of 120 kPa. The pipe B contains oil of Sp. Gravity 0.8 under a pressure of 200 kPa. The pipe A lies 2.5m above pipe B. Find the difference of pressures measured by mercury as fluid filling U-tube.

Solution:

Given : $s_a = 1.6, p_a = 120\text{kPa}; s_b = 0.8, p_b = 200\text{kPa};$

$h_1 = 2.5\text{m}$ and $s = 13.6$

Let h = Differnce of pressure measured by mercury in terms of head of water.



We know that pressure head in pipe A,

$$\frac{p_a}{\gamma} = \frac{120}{9.81} = 12.2\text{m}$$

Pressure head in pipe B, $\frac{p_b}{\gamma} = \frac{200}{9.81} = 20.4\text{m}$

We also know that pressure head in Pipe A above Z - Z

$$= 12.2 + (s_a \,.\, h_1) + s.h$$

$$= 12.2 + (1.6 \text{ x } 2.5) + 13.6 \text{ x h}$$

$$= 16.2 + 13.6 \text{ h}$$

Pressure head in Pipe B above Z - Z

$$= 20.4 + s_b h = 20.4 + (0.8 \text{ x h})$$

Equating;

$$16.2 + 13.6 \text{ h} = 20.4 + (0.8 \text{ x h})$$

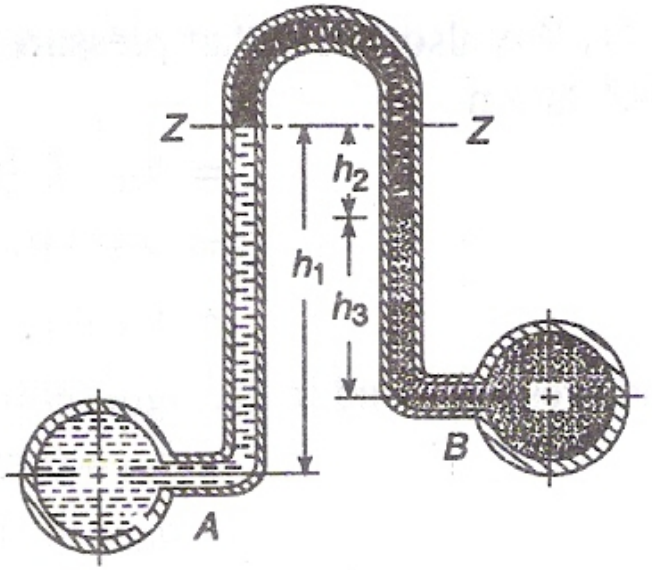
$$\text{h} = 0.328 \text{ m} = 328 \text{ mm}$$

Inverted Differential Manometer:

- Type of differential manometer in which an inverted U-tube is used.
- Used for measuring difference of low pressure.

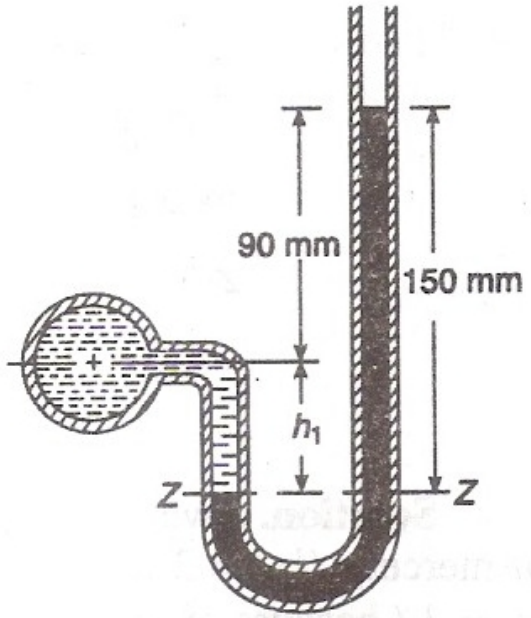
1. Pressure head in the left limb above Z-Z = ha-s1h1
2. Pressure head in the right limb above Z-Z = hb-s2h2-s3h3
3. Equating we get, ha-s1h1 = hb-s2h2-s3h3

(Where; ha, hb are Pressure in pipes A and B
expressed in terms of head of liquid, respectively)



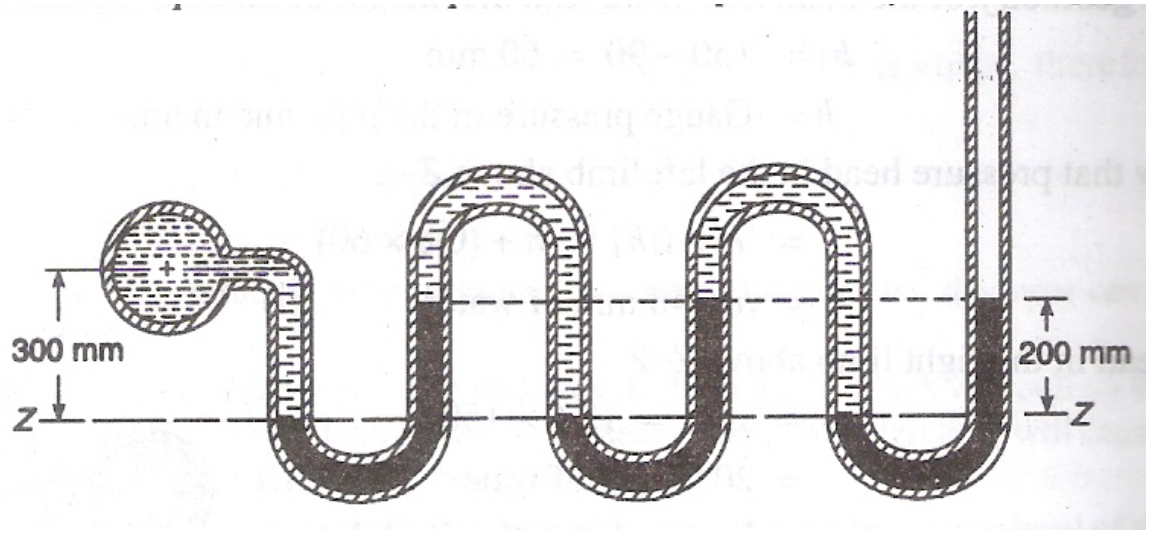
Exercise (Assignment):

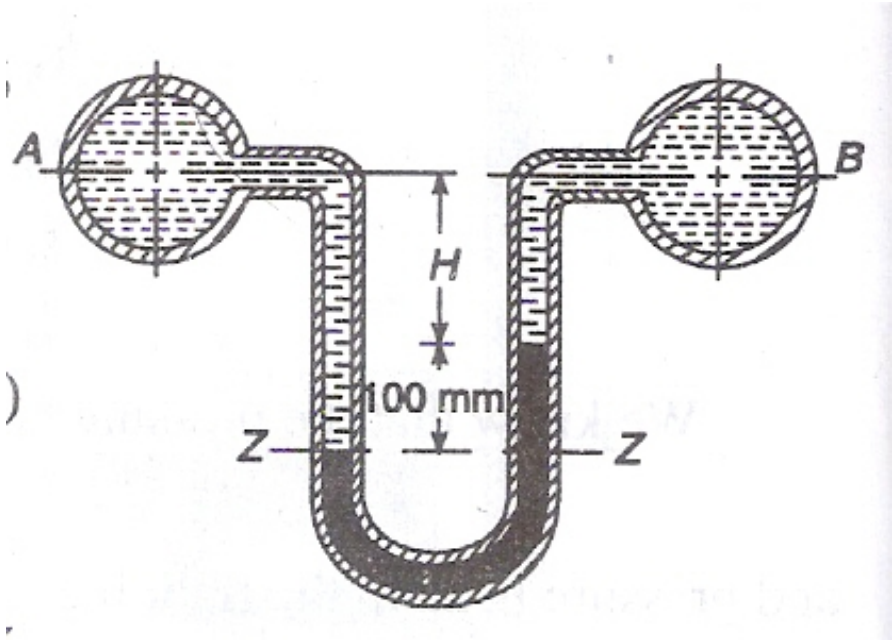
1. A simple manometer is used to measure the pressure of oil (sp. Gravity = 0.8) flowing in a pipeline. Its right limb is open to the atmosphere and the left limb is connected to the pipe. The centre of pipe is 90mm below the level of mercury (sp. Gravity = 13.6) in the right limb. If the difference of mercury levels in the two limbs is 150mm, find the pressure of oil in pipe.



Exercise (Assignment):

2. The pressure of water flowing in a pipeline is measured by a manometer containing U-tubes as shown in figure. The measuring fluid is mercury in all the tubes and water is enclosed between the mercury columns. The last tube is open to the atmosphere. Find the pressure of oil in the pipeline.



Exercise (Assignment):

3. A differential manometer connected at the two points A and B at the same level in a pipe containing an oil of Sp. Gravity 0.8, shows a difference in mercury levels as 100mm. Determine the difference in pressures at the two points.