Applications of Bernoulli’s theorem

Lecture - 7

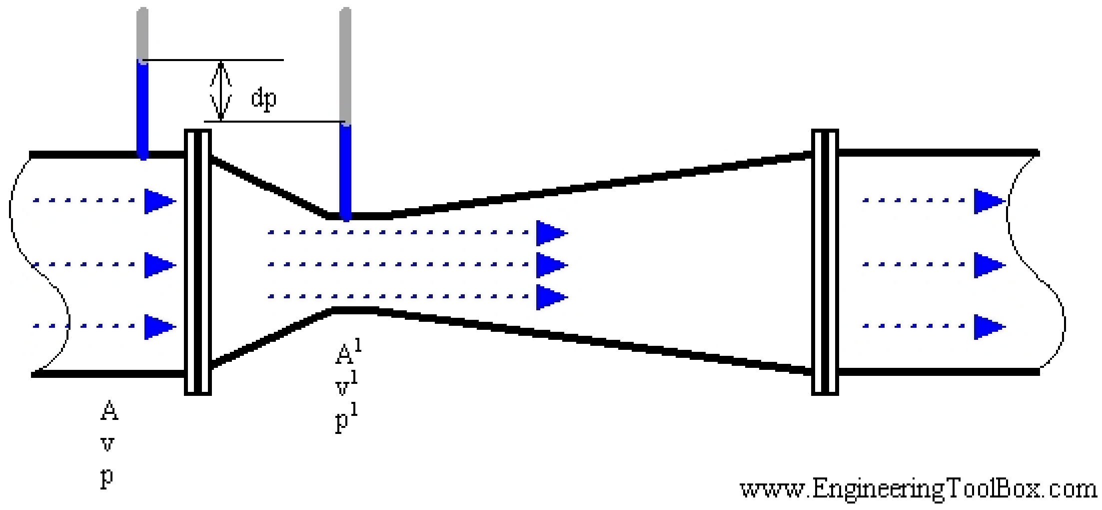
Practical Applications of Bernoulli's Theorem

- The Bernoulli equation can be applied to a great many situations not just the pipe flow we have been considering up to now.
- In the following sections we will see some examples of its application to flow measurement from tanks, within pipes as well as in open channels.

1. **Venturimeter**
2. **Orificemeter**
3. **Pitot tube**

1. Venturimeter:

- The Venturimeter is a device for measuring discharge in a pipe.
- It consists of three parts.
    a. Convergent Cone
    b. Throat
    c. Divergent Cone



a. Convergent Cone:

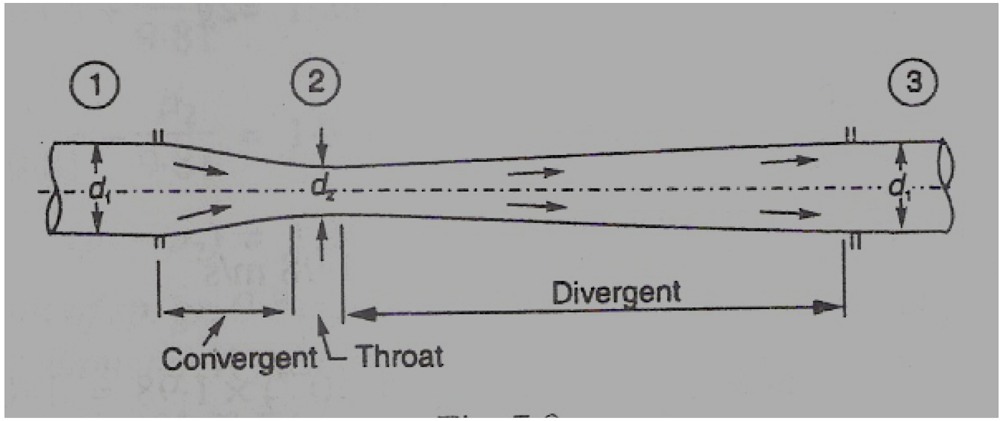
- It is a short pipe which converges from a diameter d_1 (diameter of a pipe in which a venturimeter is fitted) to a smaller diameter d_2.
- The convergent cone is also known as inlet of the venturimeter.
- The slope of the converging sides is between 1in 4 or 1in 5.

b. Throat:

- It is a small portion of circular pipe in which the diameter d_2 is kept constant.

c. Divergent Cone:

- It is a pipe, which diverges from a diameter d_2 to a large diameter d_1.
- The divergent cone is also known as outlet of venturimeter.
- The length of the divergent cone is about 3 to 4 times than that of convergent cone.



How it operates?

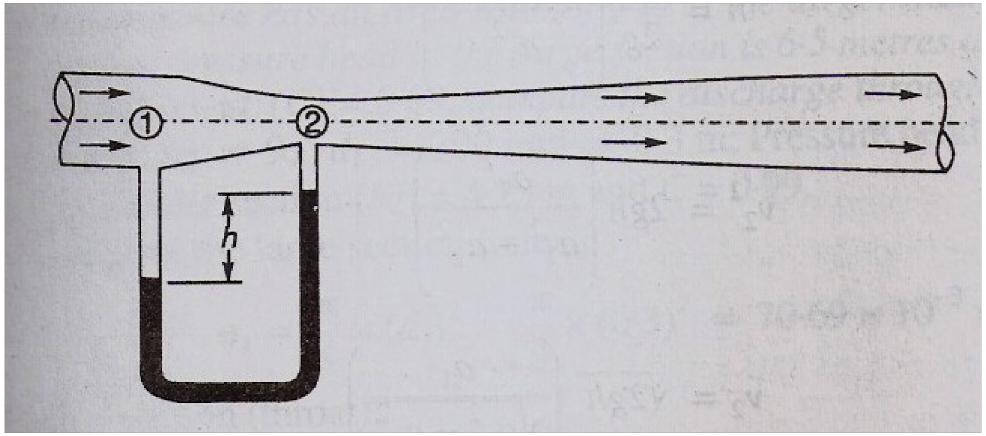
- It consists of a rapidly converging section, which increases the velocity of flow and hence reduces the pressure (acceleration b/w section 1-2).
- It then returns to the original dimensions of the pipe by a gently diverging ‘ diffuser ’ section (deceleration b/w section 2-3).
- By measuring the pressure differences the discharge can be calculated.
- This is a particularly accurate method of flow measurement as energy losses are very small.

Why the divergent cone is made longer?

- As a result of retardation (section 2-3), the velocity decreases and pressure increases.
- If the pressure is rapidly recovered, then there is every possibility for the stream of liquid to break away from the walls of meter.
- In order to avoid the tendency of breaking away the stream of liquid, the divergent cone is made sufficiently longer.
- Another reason is to minimize friction losses.
- Divergent cone is 3 to 4 times longer than convergent cone.

Measurement of Discharge:

- Consider a venturimeter through which some liquid is flowing.



Let

- p_1 = Pressure at section 1
- V_1 = Velocity of water at section 1
- z_1 = Datum head at section 1
- a_1 = Area of venturimeter at section 1
- p_2, V_2, z_2, a_2 = Corresponding values at section 2

Applying Bernoulli's equation at sections 1 and 2 i.e,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} \qquad (1)$$

Let datum line be the axis of venturimeter,

Now $z_1 = 0$ and $z_2 = 0$

$$\therefore \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

$$or \quad \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad (2)$$

Since the discharge at Section 1 & 2 is continuous, therefore

$$V_1 = \frac{a_2 V_2}{a_1} \qquad (\because a_1 V_1 = a_2 V_2)$$

$$\therefore V_1^2 = \frac{a_2^2 V_2^2}{a_1^2}$$

Substituting value in equation 2.

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{V_2{}^2}{2g} - \frac{a_2^2 V_2{}^2}{a_1{}^2 \, . \, 2g}$$

$$= \frac{V_2{}^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

We know that $\frac{p_1}{\gamma} - \frac{p_2}{\gamma}$ is the difference between the pressure heads at section 1 & 2. When the pipe is horizontal, this difference represents the venturi head and is denoded by h.

$$\text{or} \quad \text{h} = \frac{V_2{}^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

$$\text{V}_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right)$$

$$V_2 = \sqrt{2gh}\left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}}\right)$$

We know that discharge through a venturimter,

$$Q = \text{Coefficent of Venturimter} \,.\, a_2.V_2$$

$$Q = C \,.\, a_2.V_2$$

$$Q = \left(\frac{Ca_1a_2}{\sqrt{a_1^2 - a_2^2}}\right)\sqrt{2gh}$$

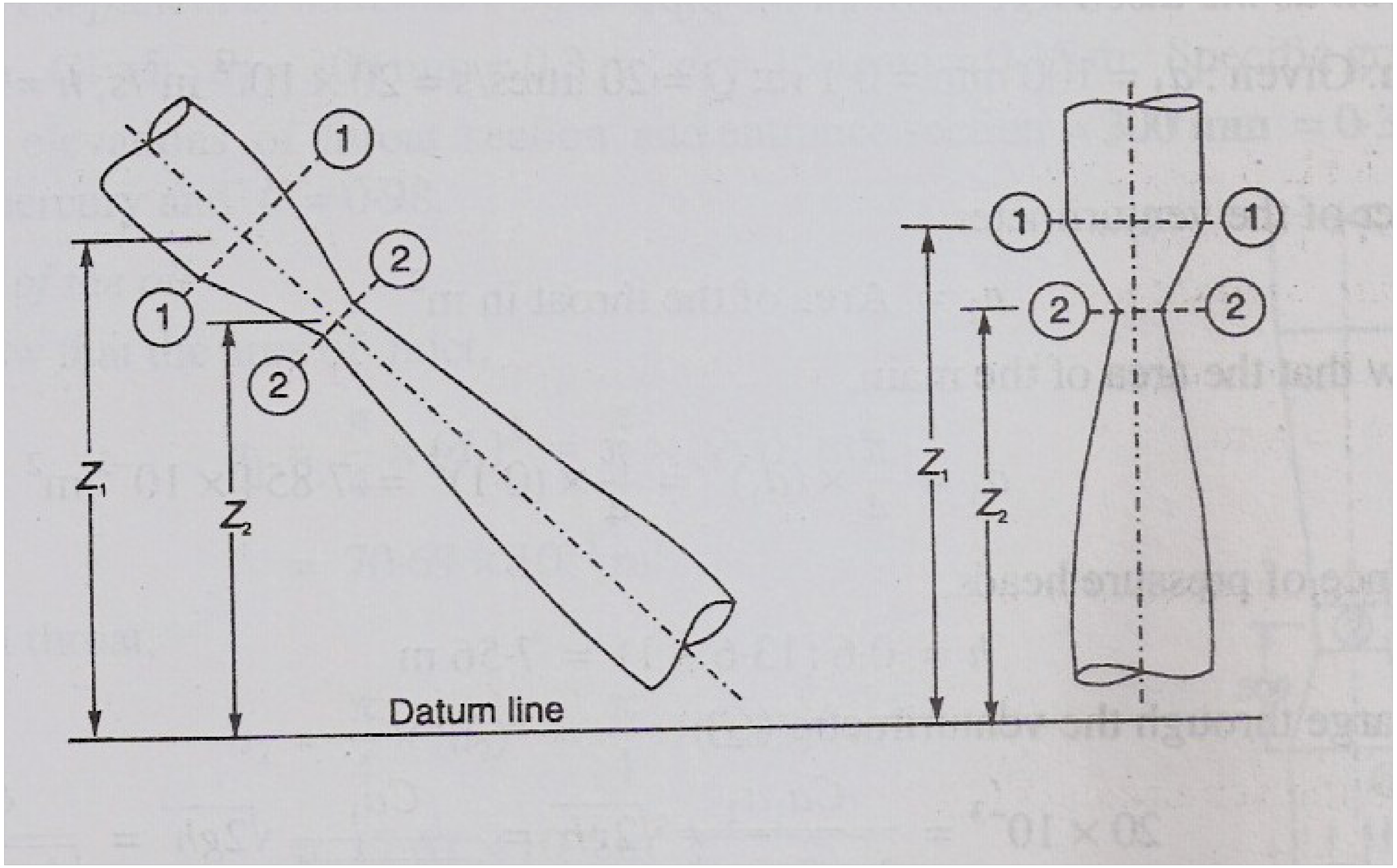
Note:

The venturi head (h), in above equation is taken in terms of liquid head. But, in actual practice, this head is given as mercury head. In such a case the mercury head should be converted into the liquid head.

h = (13.6 - s) / s   x   Head of mercury

Where, 13.6 is Sp. gravity of mercury and ‘s’ is Sp. gravity of Oil.

Inclined Venturimeter:



Problems:

1. A venturimeter with a 150mm diameter at inlet and 100mm at throat is laid with its axis horizontal and is used for measuring the flow of oil (Sp. Gravity= 0.9). The oil-mercury differential manometer shows a gauge difference of 200mm. Assume coefficient of meter as 0.98. Calculate discharge in liters per minute. (Ans, Q=3834 lit/min).
2. A venturimeter has an area ratio of 9 to 1, the larger diameter being 300mm. During the flow, the recorded pressure head in the large section in 6.5m and that at the throat 4.25m. If the meter coefficient, C=0.99, compute discharge through the meter. (Ans, 52 lit/s).
3. A horizontal venturimeter 160mm x 80mm is used to measure the flow of an oil of Sp. Gracity 0.8. Determine the deflection of the oil-mercury gauge, if the discharge of the oil is 50lit/s. Take coefficient of venturimeter as 1. (Ans, 296mm).

Problems:

4. A venturimeter is to be filled to a 250mm diameter pipe, in which the maximum flow is 7200 lit/min and the pressure head is 6m of water. What is the minimum diameter of throat, so that there is no negative head in it? (Ans, 117mm)
5. A 300mm x 150mm venturimeter is provided in a vertical pipeline carrying oil of Sp. Gravity 0.9, the flow being upwards. The difference in elevations of the throat section and entrance section of the venturimeter is 300mm. The differential U tube mercury manometer shows a gauge deflection of 250mm. Calculate

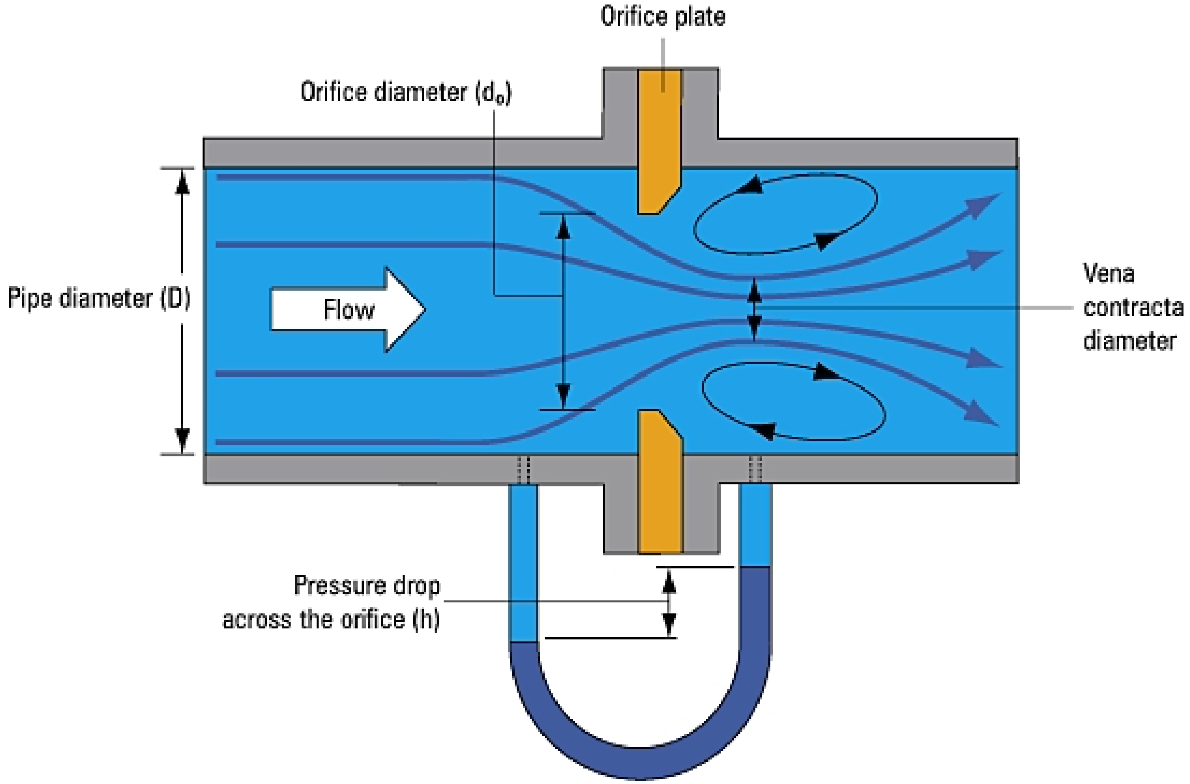
   i) discharge of the oil

   ii) pressure difference b/w the entrance and throat section.

   (Ans, i) Q = 149 lit/s ii) 3.695m)

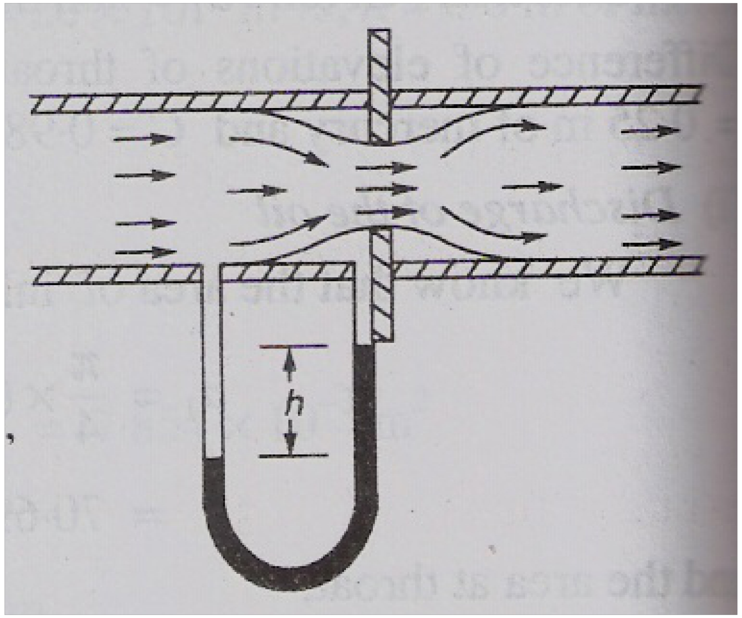
2. Orifice Meter:

- An orifice meter is used to measure the discharge in a pipe. It consists of a plate having a sharp edged circular hole known as an orifice. This plate is fixed inside a pipe.



Measurement of Discharge:

- A mercury manometer is inserted to know the difference of pressure between the pipe and the throat. (i.e., orifice)



Let

- h = Reading of mercury manometer
- p_1 = Pressure at the inlet
- V_1 = Velocity of liquid at inlet
- a_1 = Area of pipe at inlet
- p_2, V_2, a_2 = Corresponding values at throat

Applying Bernoulli's equation for inlet of pipe and the throat,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} \qquad (1)$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad (\because z_1 = z_2)$$

or $$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{1}{2g}\left(V_2^2 - V_1^2\right)$$

Since the discharge is continuous, therefore

$$V_1 = \frac{a_2 V_2}{a_1} \qquad (\because a_1 V_1 = a_2 V_2)$$

$$\therefore V_1^2 = \frac{a_2^2 V_2^2}{a_1^2}$$

Substituting value in equation 2.

$$h = \frac{1}{2g}\left(V_2^2 - \frac{a_2^2 V_2{}^2}{a_1{}^2}\right) = \frac{V_2{}^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

$$\mathrm{V}_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right)$$

$$\mathrm{V}_2 = \sqrt{2gh}\left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}}\right)$$

We know that discharge,

$$\mathrm{Q} = \text{Coefficent of Orifice Meter}\,.\ \mathrm{a}_2.V_2$$

$$\mathrm{Q} = \mathrm{C}\,.\ \mathrm{a}_2.V_2$$

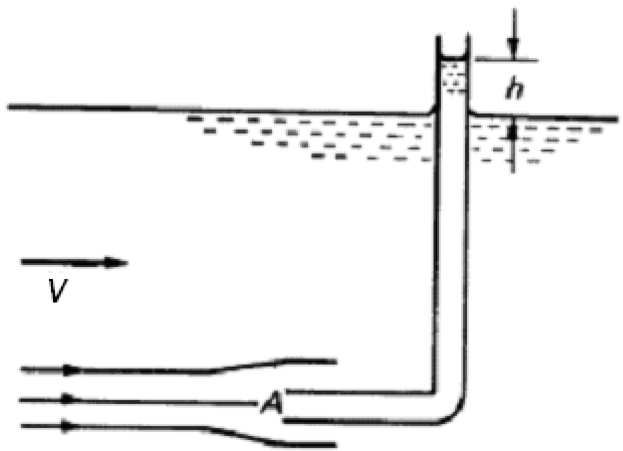
$$\mathrm{Q} = \left(\frac{Ca_1a_2}{\sqrt{a_1^2 - a_2^2}}\right)\sqrt{2gh} \qquad \text{(Same as venturimeter)}$$

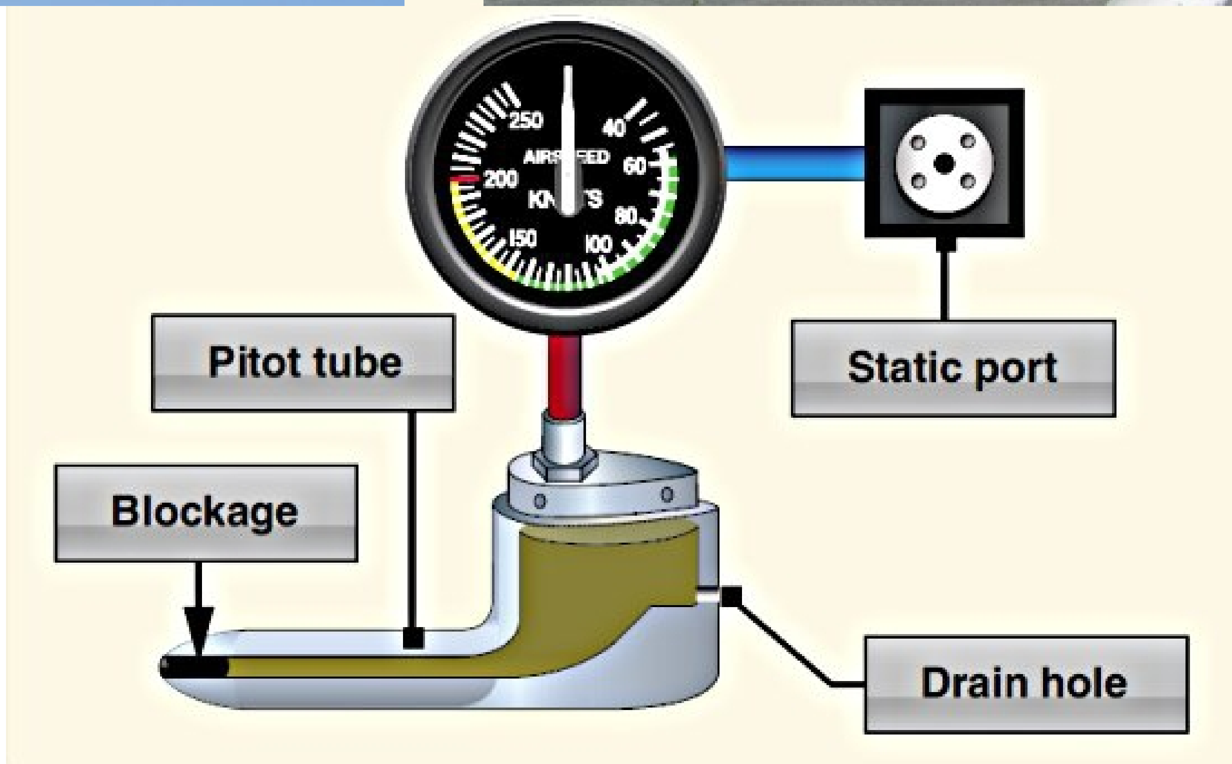
Problem:

- An orifice meter consisting of 100 mm diameter orifice in a 250mm diameter pipe has coefficient equal to 0.65. The pipe delivers oil (Sp. Gravity 0.8). The pressure difference on the two sides of the orifice plate is measured by a mercury oil differential manometer. If the differential gauge reads 80mm of mercury, calculate the rate of flow in lit/s. (Ans, 82 lit/s)

3. Pitot Tube:

- A Pitot tube is an instrument to determine the velocity of flow at the required point in a pipe or a stream.
- It consists of glass tube bent a through 90^{o}
- The lower end of the tube faces the direction of the flow.
- The liquid rises up in the tube due to the pressure exerted by the flowing liquid .
- By measuring the rise of liquid in the tube, we can find out the velocity of the liquid flow.




Pitot tube
Blockage
Static port
Drain hole

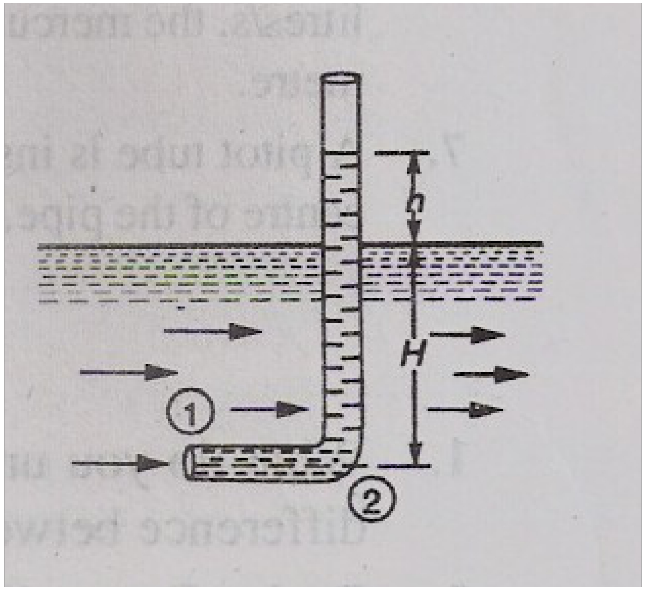
Finding Velocity:

- Let
- h = Height of liquid in the pitot tube above the surface.
- H = Depth of tube in the liquid
- V = velocity of the liquid
- Applying Bernoulli's equation for the section 1 & 2.

$$H + \frac{V^2}{2g} = \mathrm{H} + \mathrm{h}$$

$$\mathrm{h} = \frac{\mathrm{V}^2}{2\mathrm{g}}$$

$$V = \sqrt{2gh}$$



Problem:

- A pitot tube was inserted in a pipe to measure the velocity of water in it. If the water rises in the tube is 200mm. Find velocity of water. (Ans, 1.98m/s)

FLOW THROUGH ORIFICES

Measurement of Discharge

Introduction:

- "Orifice is an opening in a vessel through which the liquid flows out."
- *This hole or opening is called an orifice, so long as the level of the liquid on the upstream side is above the top of the orifice.*
- The usual purpose of an orifice is the measurement of discharge.
- It can be provided in the vertical side of the vessel on in the base. But the former is more common.

Types of Orifices According to:

Size

- Small
- Large

Shape

- Circular
- Rectangular
- Triangular

Shape of the edge

- Sharp-edged
- Bell-mouthed

Nature of Discharge

- Fully submerged
- Partially submerged

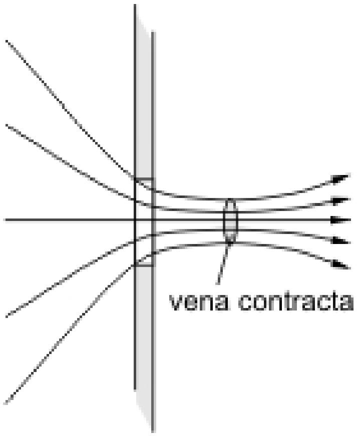
Important Terms:

- Jet of Water:

  "The continuous stream of liquid, that comes out or flows out of an orifice, is known **as Jet of water**."

- Vena Contracta:

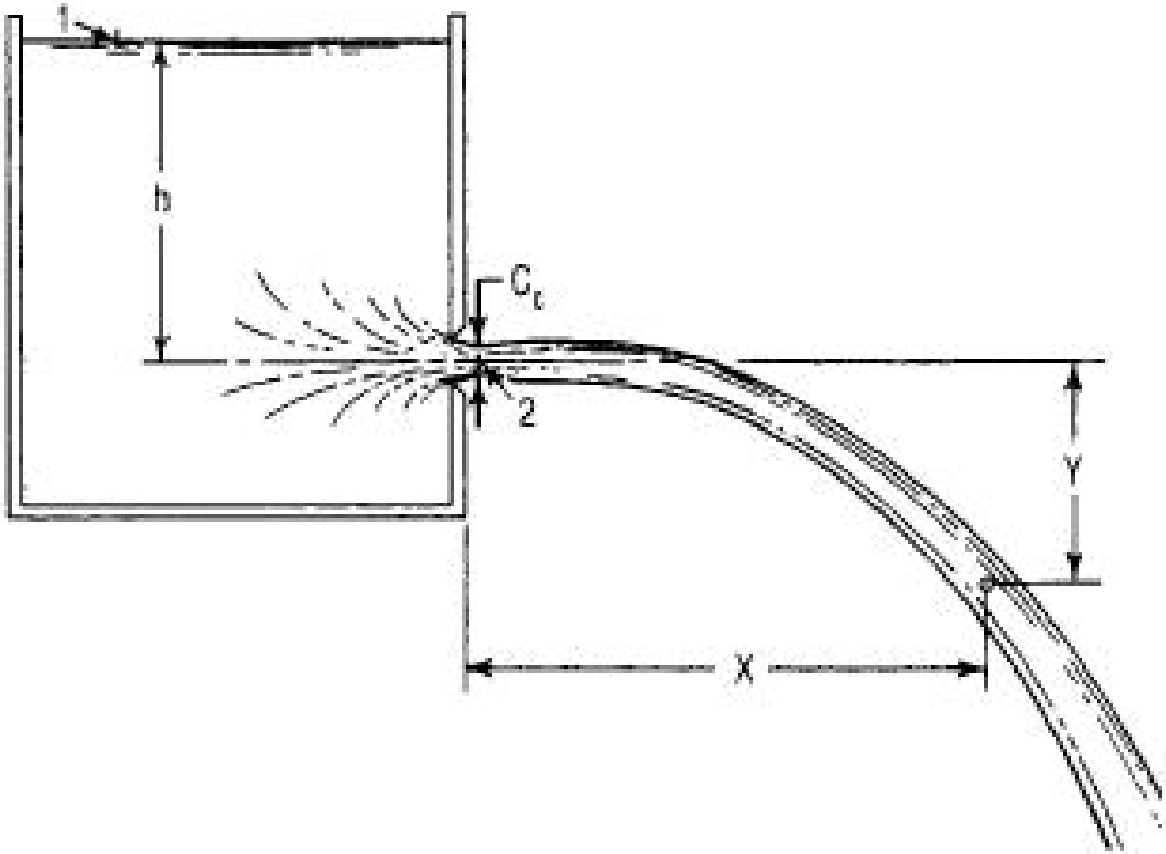
- **Vena contracta** is the point in a fluid stream where the diameter of the stream is the least, and fluid velocity is at its maximum.

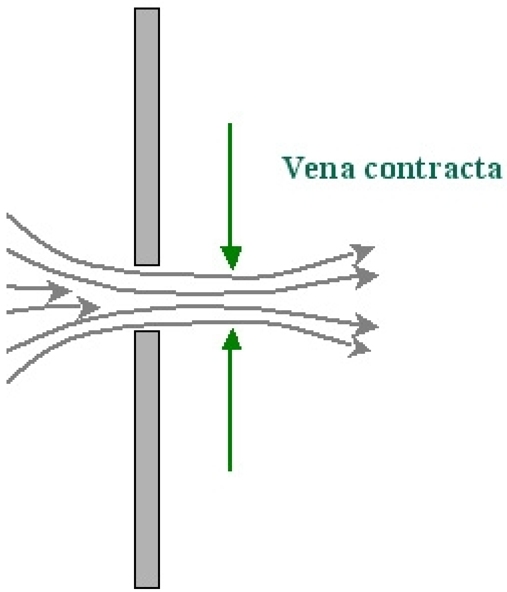


Vena Contracta:

- Consider a tank, fitted with an orifice. The liquid particle, in order to flow out through the orifice, move towards the orifice from all directions.
- A few of the particles first move downward, then take a turn to enter into the orifice and then finally flow through it.
- It may be noted, that the liquid particles lose some energy, while taking the turn to enter into the orifice.
- It has been thus observed that the jet, after leaving the orifice, gets contracted.
- The maximum contraction takes place at a section slightly on the downstream side of the orifice, where the jet is more or less horizontal. Such a section is known as vena contracta as shown by section C (1-2) in figure.

Vena Contracta:





Hydraulic Coefficients:

Following four coefficients are known as hydraulic coefficients or orifice Coefficient.

1) Coefficient of contraction
2) Coefficient of velocity
3) Coefficient of discharge
4) Coefficient of resistance

1. Coefficient of Contraction:

- “ The ratio of area of jet, at vena contracta, to the area of orifice is known as coefficient of contraction.”
- Mathematically,

$$C_c = \frac{\text{Area of jet at vena Contracta}}{\text{Area of Orifice}}$$

- The value varies slightly with the available head of the liquid, size and the shape of the orifice.
- ***An average value of C_c is about 0.64***.

2. Coefficient of Velocity:

- "The ratio of actual velocity of the jet, at vena contracta, to the theoretical velocity is known as coefficient of velocity."
- Mathematically,

$$C_v = \frac{\text{Actual velocity of jet at vena Contracta}}{\text{Theoretical velocity of jet}}$$

- The difference between the velocities is due to friction of the orifice.
- The value of coefficient of velocity varies slightly with the different shapes of the edges of the orifices.
- For a sharp edged orifice, the value of C_v increases with the head of water.

2. Coefficient of Velocity:

- The following table gives the values of C_v for an orifice of 10mm diameter with the corresponding head (given by Weisback).

H	20mm	500mm	3.5m	20m	100m
C_v	0.959	0.967	0.975	0.991	0.994

Note:

- ***An Average value of C_v is about 0.97.***
- The ***theoretical velocity*** of jet at vena contracta is given by relation :

$$V = \sqrt{2gh}$$

Where, h is head of water at vena contracta.

3. Coefficient of Discharge:

- "It is the ratio of actual discharge through an orifice to the theoretical discharge."
- Mathematically,

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

$$= \frac{\text{Actual velocity x Actual area}}{\text{Theoretical velocity x Theoretical area}}$$

$$= C_v \text{ x } C_c$$

- ***Average value of coefficient of discharge varies from 0.60 to 0.64.***

4. Coefficient of Resistance:

- "The ratio of loss of head in the orifice to the head of water available at the exit of the orifice is known as coefficient of resistance."
- Mathematically,

$$C_r = \frac{\text{Loss of head in the orifice}}{\text{Head of water}}$$

- The loss of head in the orifice takes place, because the walls of the orifice offer some resistance to the liquid as it comes out.
- *The coefficient of resistance is generally neglected, while solving numerical.*

Problems:

1. A jet of water issues from an orifice of diameter 20mm under a head of 1m. What is the coefficient of discharge for the orifice, if actual discharge is 0.85lit/s. (Ans, 0.61)
2. A 60mm diameter orifice is discharging water under a head of 9m. Calculate the actual discharge through the orifice in Lit/s and actual velocity of the jet in m/s at vena contracta, if $C_d = 0.625$ and $C_v = 0.98$. (Ans, Q = 23.5 lit/s & V_{ac} = 13m/s)