Energy Consideration in Steady Flow

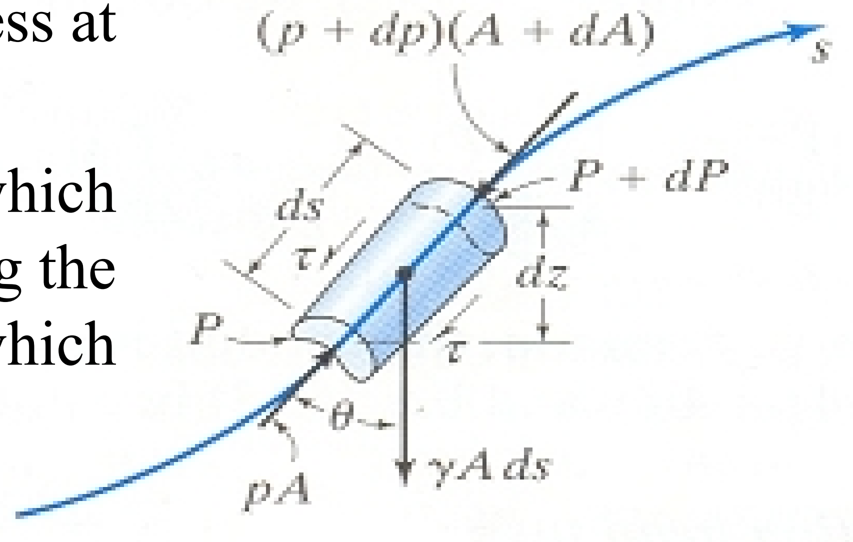
Lecture - 8

EQUATION FOR STEADY MOTION OF A REAL FLUID ALONG A STREAMLINE

- Following the same procedure as in the previous section, except that now we shall consider a real fluid. The real fluid element in a stream tube depicted in Fig. 5.2 is similar to that of Fig. 5.1, except that now with the real fluid there is an additional force acting because of fluid friction.

$$\tau(P + 1/2dP)ds$$

- where τ (tau) is the shear stress at the boundary of the element and
- (P + 1/2dP)ds is the area over which the shear stress acts, P being the perimeter of the end area A, which may have any shape.



- Writing ƐF = ma along the streamline and neglecting second-order terms, for steady flow we now get:

$$-dpA-\rho gAdz-\tau Pds=(\rho dsA)V\frac{dV}{ds}$$

- Dividing through by *pA* and rearranging gives

$$\frac{dp}{\rho}+gdz+VdV=-\frac{\tau P}{\rho A}ds \qquad (5.10)$$

- As before, we can also express as:

$$\frac{dp}{\gamma}+dz+d\frac{V^2}{2g}=\frac{\tau P}{\gamma A}ds \qquad (5.11)$$

- These equations apply to steady flow of both compressible and incom-pressible real fluids.

Incompressible Fluid:

- For an incompressible fluid (γ = constant), we can integrate Eq. (5.11) directly. Integrating from some point 1 to another point 2 on the same streamline, where the distance between them is *L,* we get for an incompressible real fluid:

$$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = -\frac{\tau PL}{\gamma A}$$

- Or **Energy per unit weight:**

$$\left(\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right) - \frac{\tau PL}{\gamma A} = \left(\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right) \qquad (5.12)$$

Assumptions:

1. Steady flow
2. Incompressible fluid
3. Along a streamline
4. No energy added or removed

Head:

- If we compare Eq. (5.12) with Bernoulli Eq. (5.7) for ideal flow we see again the only difference is the additional term $-\tau PL/(\gamma A)$, which represents the loss of energy per unit weight due to fluid friction between points 1 and 2.
- The dimensions of this energy loss term are length only, which agrees with all the other terms in Eq. (5.12), and so this term is a form of head.

Wall friction head loss:

- The friction causing this loss of energy occurs over the boundary or surface of the element, of area *PL*. When, as occurs often, we consider the stream tube to fill the conduit, pipe, or duct conveying the fluid, *PL* becomes the inside surface area of the conduit wall, and τ becomes the shear stress at the wall, τ_o. Then we can call this energy loss term the
- Wall friction head loss:

$$h_f = \frac{\tau_0 PL}{\gamma A} \qquad (5.13)$$

- Energy per unit weight:

$$\left(\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right) - h_f = \left(\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right) \qquad (5.14)$$

Pipe friction head loss:

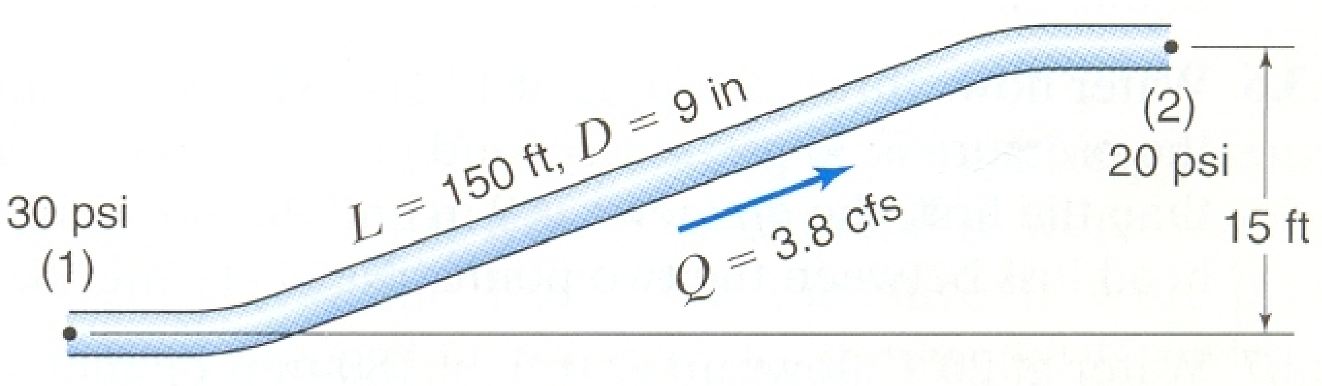
- If, as is most common, the conduit is a circular pipe of diameter *D*, then $P / A = \pi D /(\pi D^2 / 4) = 4 / D$,and Eq. (5.13) becomes the
- Pipe friction head loss:

$$h_f = \frac{4\tau_0 L}{\gamma D} \qquad (5.15)$$

- Fluid friction loss from *any* such cause, *including* wall or pipe friction, we commonly refer to as *head loss*, denoted by h_L. So wall friction head loss is usually a part of, but it may be all of, the total head loss. In a given conduit, then $h_L \geq h_f$.

Problem

- Water flows through a 150-ft-long, 9-in-diameter pipe at 3.8 cfs. At the entry point, the pressure is 30 psi; at the exit point, 15 ft higher than the entry point, the pressure is 20 psi. Between these two points, find *(a)* the pipe friction head loss, *(b)* the wall shear stress, and *(c)* the friction force on the pipe.



Solution:

(a) From Eq. (5.14):

$$h_f = \left(\frac{30(144)}{62.4} + 0 + \frac{V^2}{2g}\right) - \left(\frac{20(144)}{62.4} + 15 + \frac{V^2}{2g}\right)$$

$V_1 = V_2$ so terms in V cancel, and

$$h_f = 8.08 ft$$

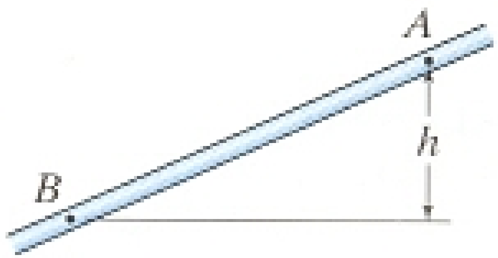
(b) From Eq. (5.15):

$$r_0 = \frac{h_f \gamma D}{4L} = \frac{8.08(62.4)0.75}{4(150)} = 0.630 lb/ft^2$$

(c) Friction Force $= r_0 PL = r_0(\pi D)L = 0.630\pi(0.75)150 = 223 lb$

Exercise

1. **(5.3.1)** A vertical pipe of 4 ft diameter and 60 ft long has a pressure head of 22.7 ft of water at its upper end. When the flow of water through it is such that the mean velocity is 16 fps, the pipe friction head loss is hf = 2.8 ft. Find the pressure head at the lower end of the pipe when the flow is (a) downward; (b) upward.
2. **(5.3.4)** In Fig. the pipe *AB* is of uniform diameter and h = 28 ft. The pressure at *A* is 30 psi and at *B* is 40 psi. In which direction is the flow, and what is the pipe friction head loss in feet of the fluid if the liquid has a specific weight of *(a)* 35 Ib/ft³, *(b)* 92 Ib/ft³?



Exercise

3. **(5.3.6)** Water flows through a pipe at 14 cfs. At a point where the pipe diameter is 18 in, the pressure is 30 psi; at a second point, further along the flow path and 2 ft lower than the first, the diameter is 9 in and the pressure is 18 psi. Find the pipe friction head loss between the two points. Neglect other head losses.
4. **(5.3.7)** Water at 20 °C flows up a straight 180-mm-diameter pipe that slopes at 12° to the horizontal. Find the shear stress at the wall, if the pressure is 100 kPa at point 1, and 25 kPa at higher point 2 that is 30 m further along the pipe.

HEAD

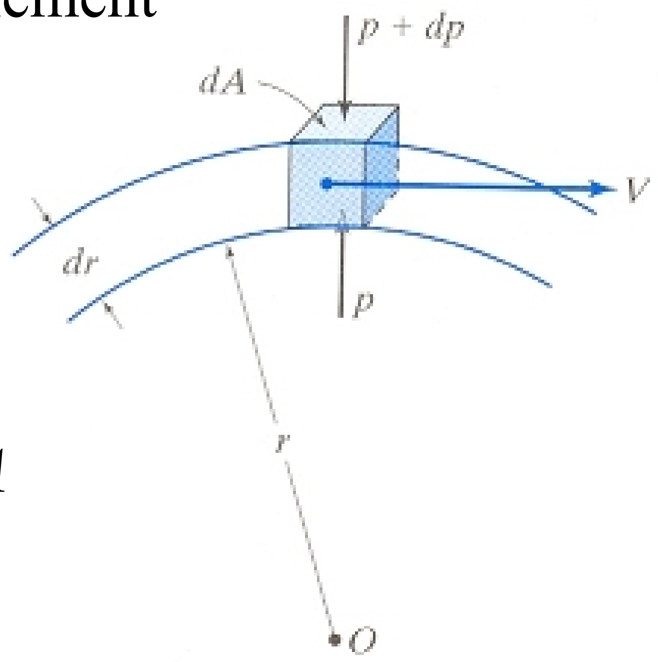
$$\left(\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}\right)-h_L=\left(\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}\right) \qquad (5.28)$$

- In above equation each term has the dimensions of length. Thus p/γ, called the pressure head, represents the energy per unit weight stored in the fluid by virtue of the pressure under which the fluid exists.
- Z called the elevation head or potential head, represents the potential energy per pound of fluid;
- $V^2/2g$, called the velocity head, represents the kinetic energy per pound of fluid.
- We call the sum of these three terms the total head, usually denoted by H, so that

$$H=\frac{p}{\gamma}+z+\frac{V^2}{2g} \qquad (5.35)$$

FLOW IN A CURVED PATH

- The energy equations we developed previously apply fundamentally to flow along a streamline or along a stream of large cross section if we use certain average values.
- Now we will investigate conditions in a direction normal to a streamline. Figure 5.14 represents an element of fluid moving in a *horizontal* planes with a velocity V along a curved path of radius r.



- The element has a linear dimension dr in the plane of the paper and an area dA normal to the plane of the paper.

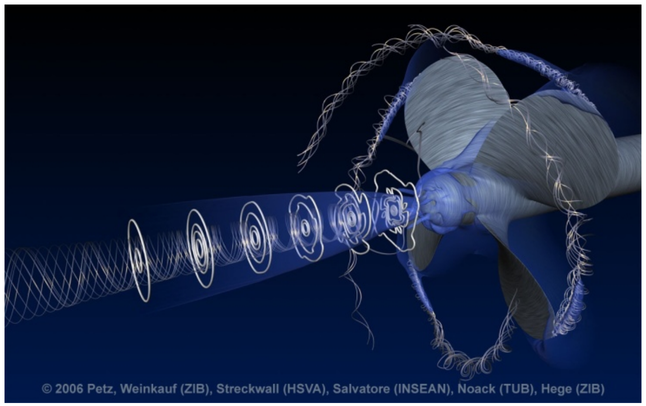
- The mass of this fluid element is $\rho dAdr$, and the normal component of acceleration is V^2/r. Thus the centripetal force acting on the element to - ward the center of curvature is $\rho dAdrV^2/r$.
- As the radius increases from r to $r + dr$, the pressure will change from p to $p + dp$. Therefore the resultant force in the direction of the center of curvature is $dpdA$. Equating these two forces and dividing by dA,

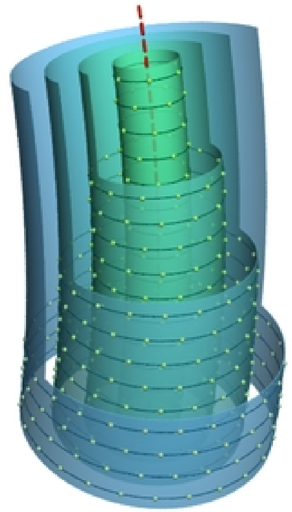
$$dp = \rho \frac{V^2}{r} dr \qquad (5.47)$$

- When horizontal flow is in a straight line for which *r* is infinity, the value of *dp* is zero. That is, no difference in pressure can exist in the horizontal direction perpendicular to horizontal flow in a straight line.
- Because *dp* is positive if *dr* is positive, Eq. (5.47) shows that pressure increases from the concave to the convex side of the stream, but the exact way in which it increases depends on the way in which *V* varies with the radius. If we can express V as a function of *r,* or if *V* is constant, we can integrate Eq. (5.47) to find $p_{outer} - p_{inner}$. Usually *V* is *not* constant.

Vortex

- *In fluid dynamics, a vortex is a region within a fluid where the flow is mostly a spinning motion about an imaginary axis, straight or curved. That motion pattern is called a vortical flow.*
- Some common examples are smoke rings, the whirlpools often seen in the wake of boats and paddles, and the winds surrounding hurricanes, tornadoes and dust devils.





Vortex Flow:

- If we take a cylindrical vessel, containing some liquid, and start rotating it, about its vertical axis, we see that the liquid will also start revolving along with the vessel.
- After some time, we shall see that the liquid surface no longer remains level. But it has been depressed down at the axis of its rotation and has risen up near the wall of the vessel on all sides.
- This type of flow, in which a liquid flows continuously round a curved path about a fixed axis of rotation is called vortex flow.

Types:

1. Forced or Rotational Flow
2. Free or Ir-rotational Flow

- Vortices are a major component of turbulent flow. In the absence of external forces, viscous friction within the fluid tends to organize the flow into a collection of so-called irrotational vortices. Within such a vortex, the fluid's velocity is greatest next to the imaginary axis, and decreases in inverse proportion distance from it. The vorticity (the curl of the fluid's velocity) is very high in a core region surrounding the axis, and nearly zero in the rest of the vortex; while the pressure drops sharply as one approaches that region.
- Once formed, vortices can move, stretch, twist, and interact in complex ways. A moving vortex carries with it some angular and linear momentum, energy, and mass. In a stationary vortex, the streamlines and pathlines are closed. In a moving or evolving vortex the streamlines and pathlines are usually spirals.