Slope-Deflection Method

Frames

Theory of Structures-II

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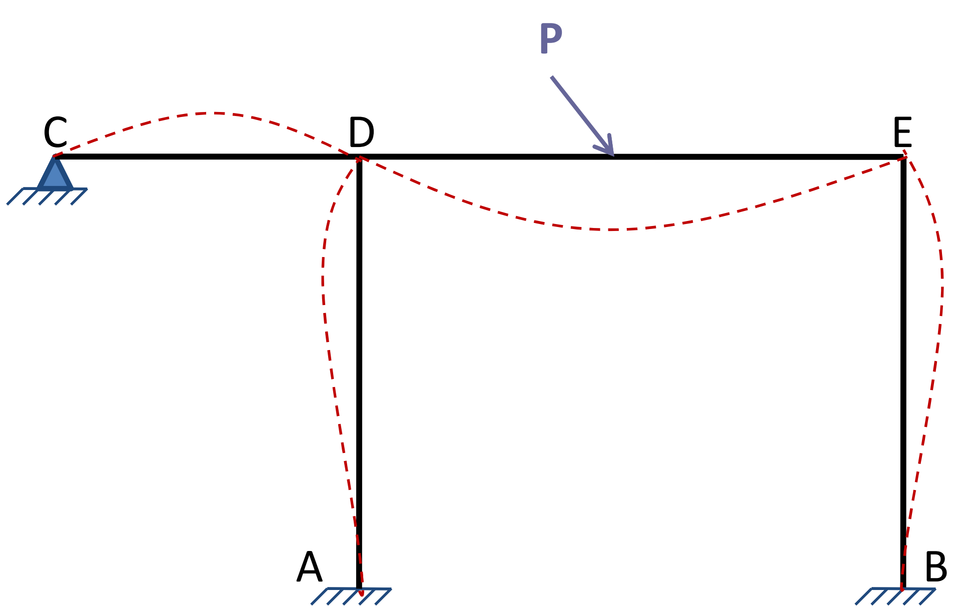
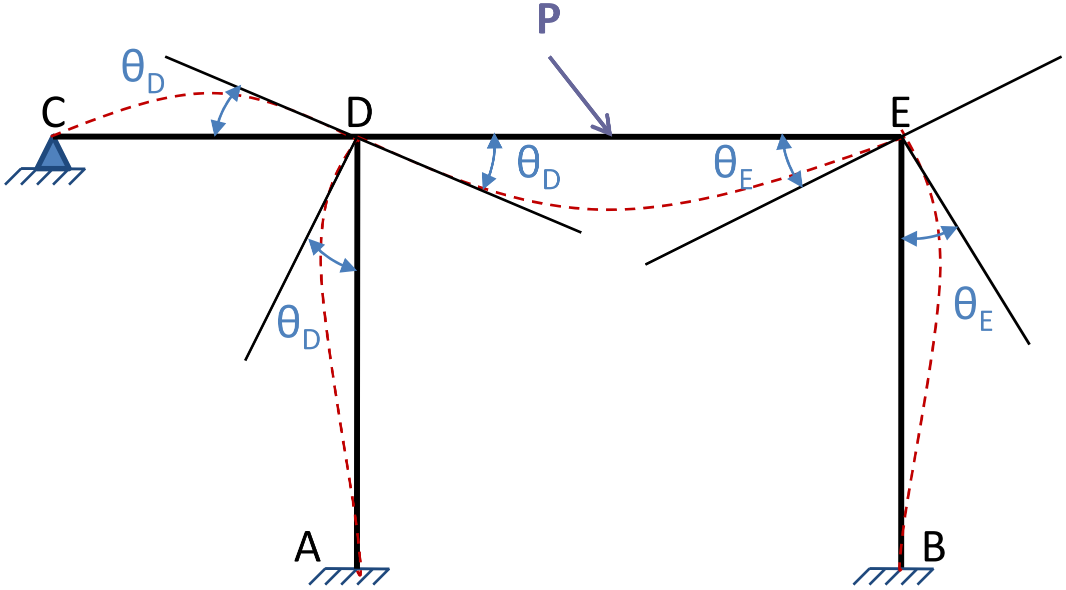
Analysis of Frames Without Sidesway

- The slope-deflection method can also be used for the analysis of frames.
- Axial deformations are neglected as they are very small.
- Consider the frame shown.



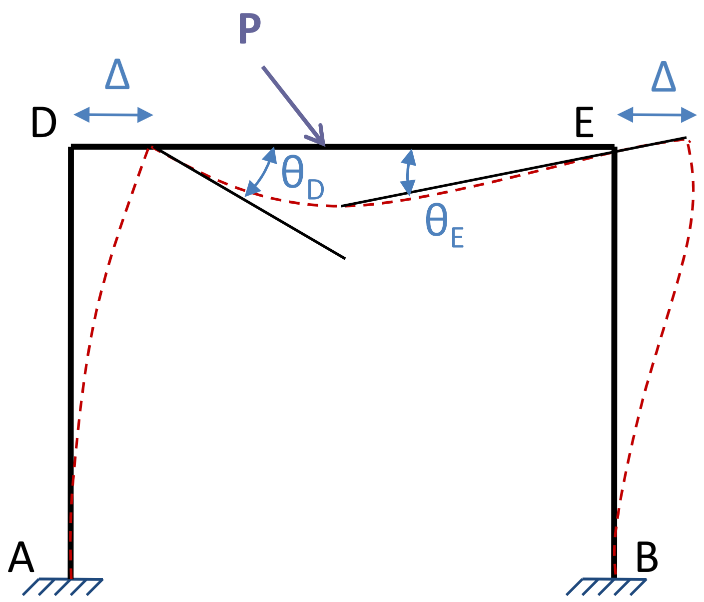
Frame without sidesway

- Fixed joint A and B can neither rotate or translate.
- Joint C can rotate but cannot translate.
- Joint D is free to rotate but its translation in any direction is prevented by member AD and CD (inextensible member).
- Joint E is free to rotate but its translation is prevented by member DE and BE.



Frame without sidesway

- Suppose that we remove member CD from the frame.



Frame with sidesway

- Joints D and E cannot translate in the vertical direction.
- But they can rotate and translate in the horizontal direction.
- The lateral displacements of building frames are known as frames with Sidesway.

- And without joint translations are known as frames without Sidesway.
- For applying SDE, distinguish should be made between two types.

Sidesway Degree of Freedom

$$ss = 2j - [2(f + h) + r + m]$$

j = number of joints

f = number of fixed supports

h = number of hinged supports

r = number of roller supports

m = number of inextensible members

Sidesway Degree of Freedom

j = 5

f = 2

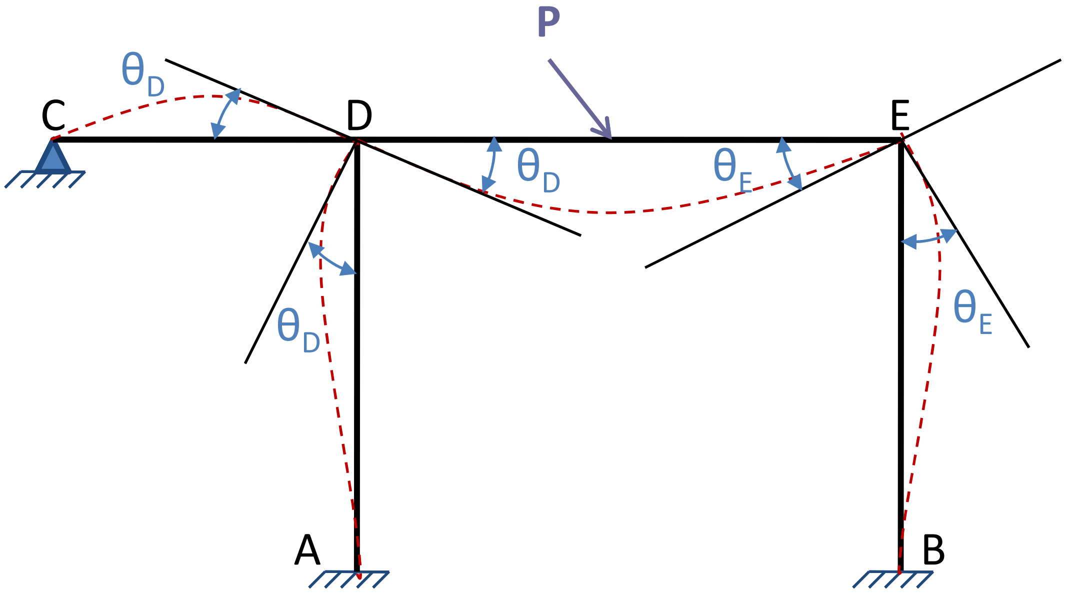
h = 1

r = 0

m = 4

$$ss = 2j - [2(f + h) + r + m]$$

$$ss = 2(5) - [2(2+1) + 4] = 0$$



Sidesway Degree of Freedom

j = 4

f = 2

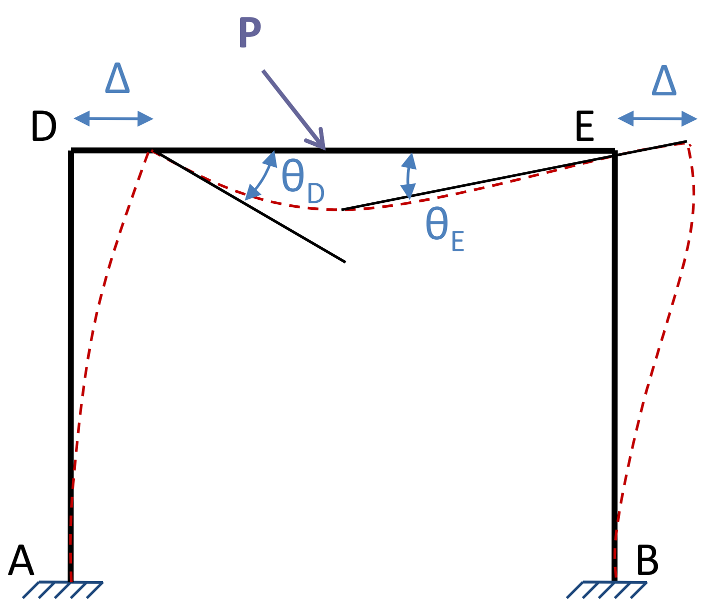
h = 0

r = 0

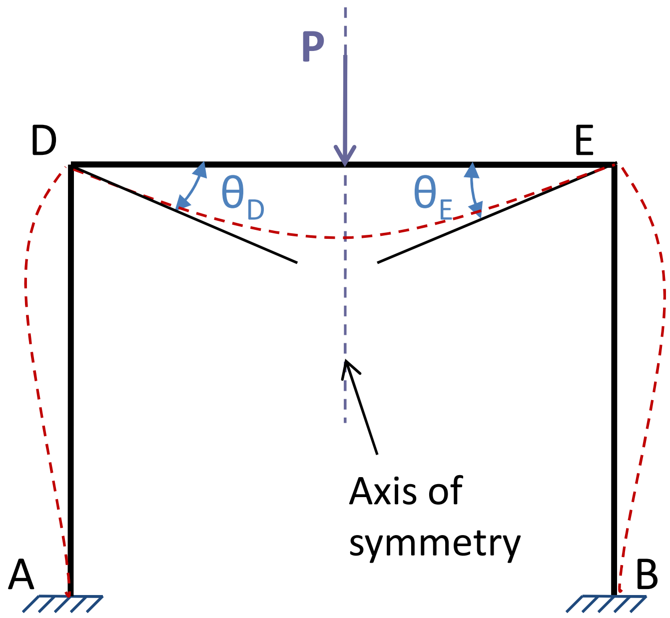
m = 3

$$ss = 2j - \left[2\left(f + h\right) + r + m\right]$$

$$ss = 2\left(4\right) - \left[2\left(2\right) + 3\right] = 1$$



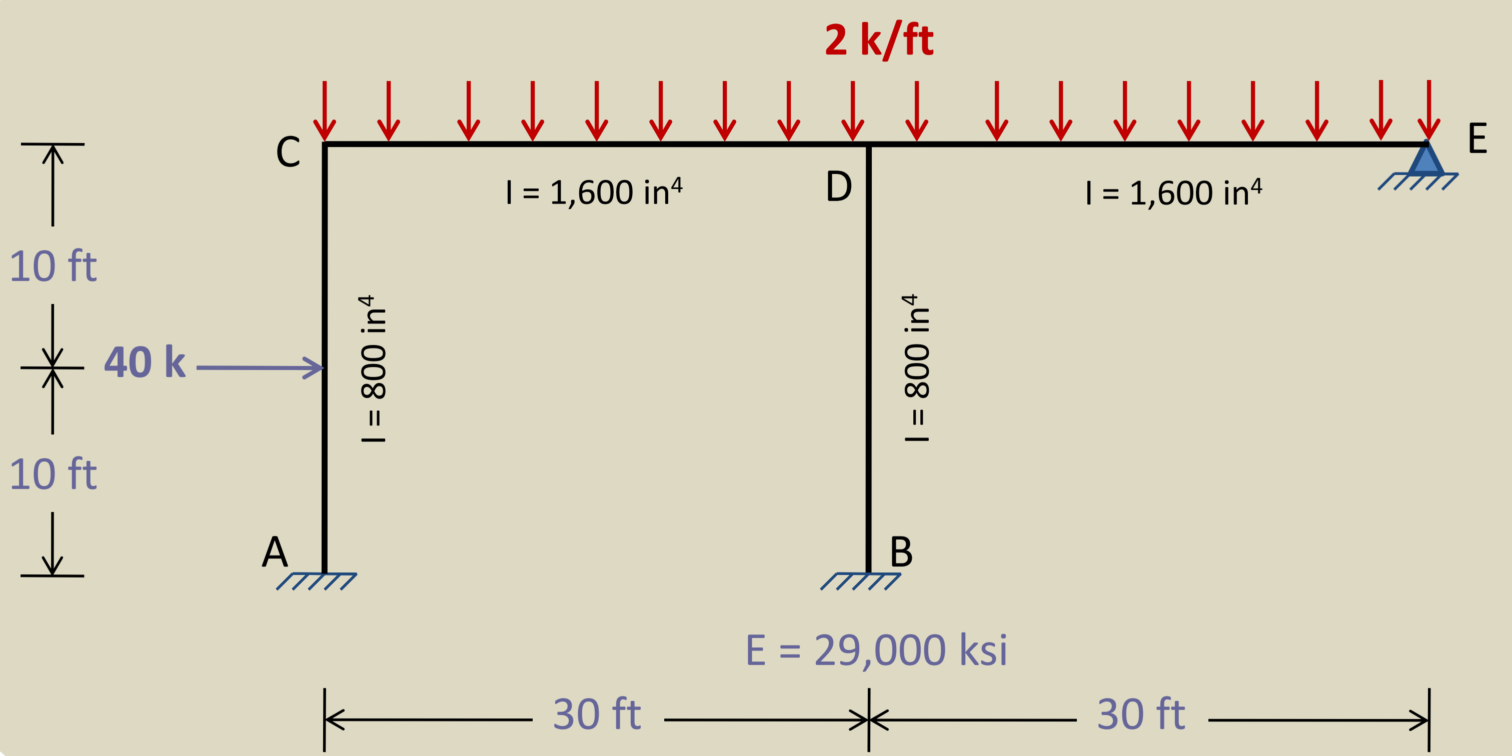
Frame with sidesway



Symmetric Frame Subjected to Symmetric Loading
No sidesway

Example 1

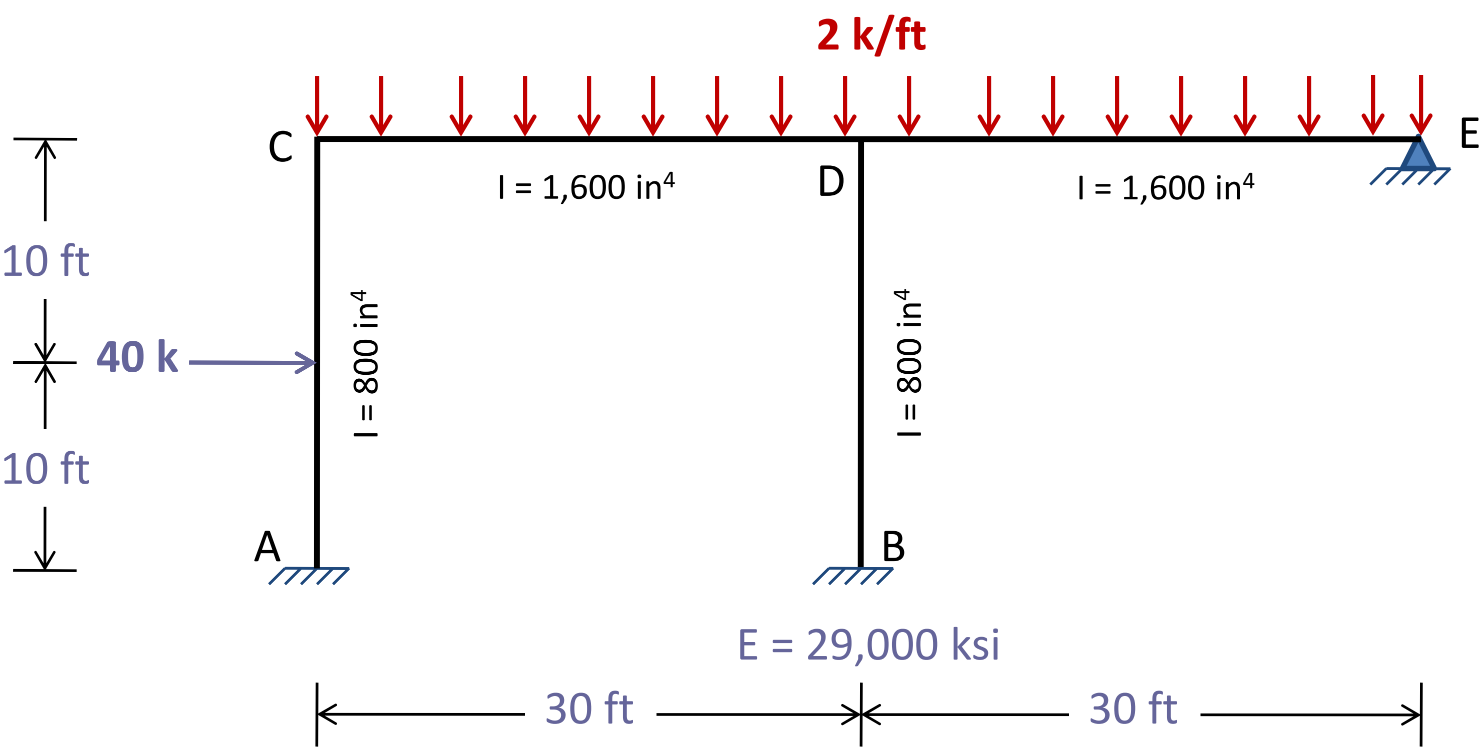
- Determine the member end moments and reactions for the frame shown by the slope-deflection method.



Solution

1. Degrees of Freedom

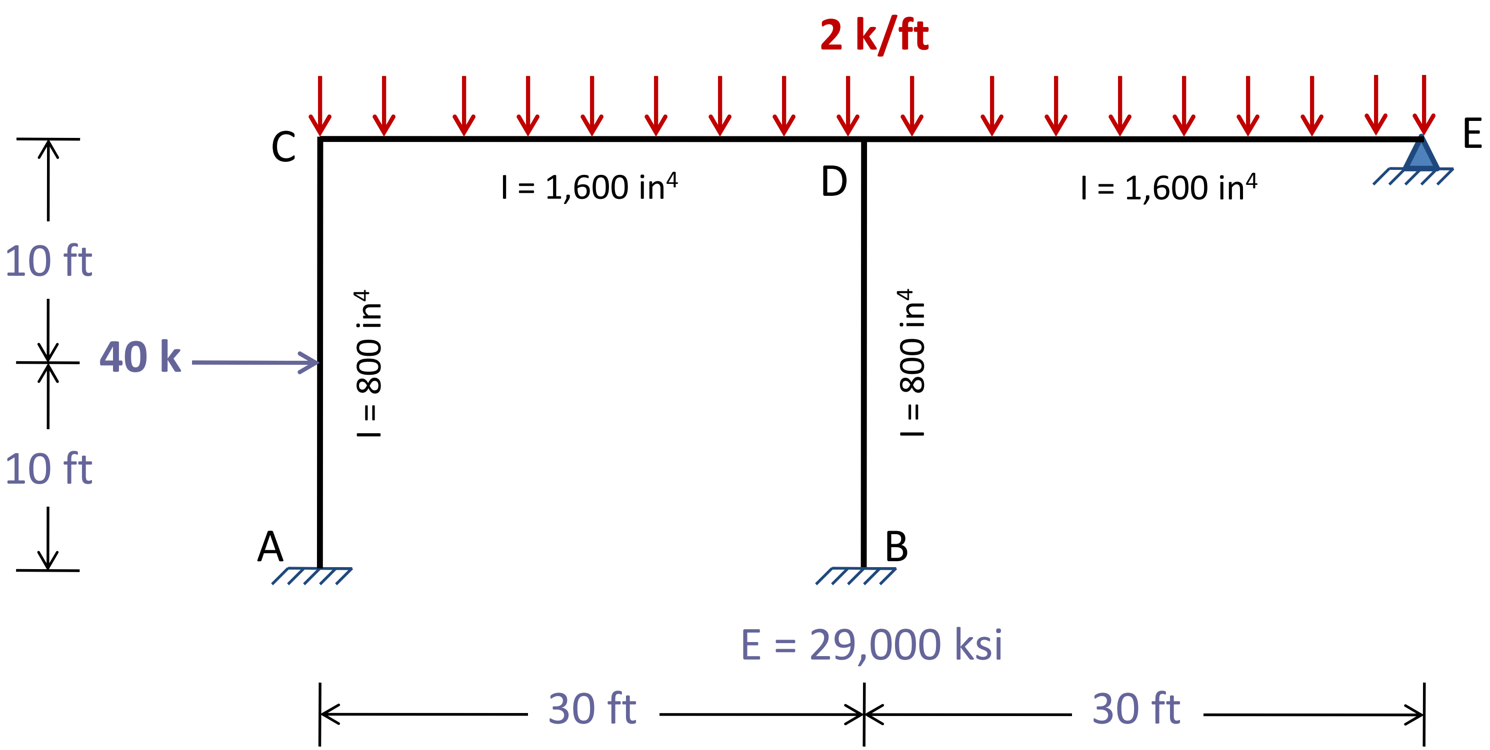
- Joints C, D and E are free to rotate.
- We will eliminate the rotation of the simple support at end E by using the modified slope-deflection equations for member DE.



Solution

1. Degrees of Freedom

- The analysis know involves two unknown joint rotations, θ_C and θ_D.



2. Fixed End Moments

- Use the expressions on the inside back cover of the book

$$FEM_{AC} = \frac{40(20)}{8} = 100\ k-ft \qquad or \qquad +100\ k\text{-}ft$$

$$FEM_{CA} = 100\ k-ft \qquad or \qquad -100\ k\text{-}ft$$

$$FEM_{BD} = FEM_{DB} = 0$$

$$FEM_{CD} = FEM_{DE} = \frac{2(30)^2}{12} = 150\ k-ft \qquad or \qquad +150\ k\text{-}ft$$

$$FEM_{DC} = FEM_{ED} = 150\ k-ft \qquad or \qquad -150\ k\text{-}ft$$

3. Slope-deflection Equations

$$I_{girder} = 1{,}600 = 2(800) = 2I$$

- We write slope-deflection equations by applying Eq. 9 to members AC, BD, and CD and Eq. 15 to member DE.

$$M_{nf} = \frac{2EI}{L}\left(2\theta_n + \theta_f - 3\psi\right) + FEM_{nf} \qquad (9)$$

$$M_{rh} = \frac{3EI}{L}\left(\theta_r - \psi\right) + \left(FEM_{rh} - \frac{FEM_{hr}}{2}\right) \qquad (15a)$$

$$M_{hr} = 0 \qquad (15b)$$

3. Slope-deflection Equations

$$M_{AC} = \frac{2EI}{20}(\theta_C) + 100 = 0.1EI\theta_C + 100 \qquad (1)$$

$$M_{CA} = \frac{2EI}{20}(2\theta_C) - 100 = 0.2EI\theta_C - 100 \qquad (2)$$

$$M_{BD} = \frac{2EI}{20}(\theta_D) = 0.1EI\theta_D \qquad (3)$$

$$M_{DB} = \frac{2EI}{20}(2\theta_D) = 0.2EI\theta_D \qquad (4)$$

$$M_{CD} = \frac{2E(2I)}{30}(2\theta_C + \theta_D) + 150 = 0.267EI\theta_C + 0.133EI\theta_D + 150 \qquad (5)$$

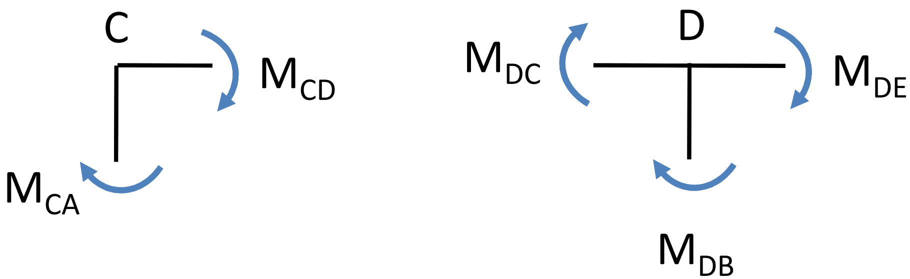
$$M_{DC} = \frac{2E(2I)}{30}(2\theta_D + \theta_C) - 150 = 0.133EI\theta_C + 0.267EI\theta_D - 150 \qquad (6)$$

$$M_{DE} = \frac{3E(2I)}{30}(\theta_D) + \left(150 + \frac{150}{2}\right) = 0.2EI\theta_D + 225 \qquad (7)$$

$$M_{ED} = 0 \qquad \text{ANS}$$

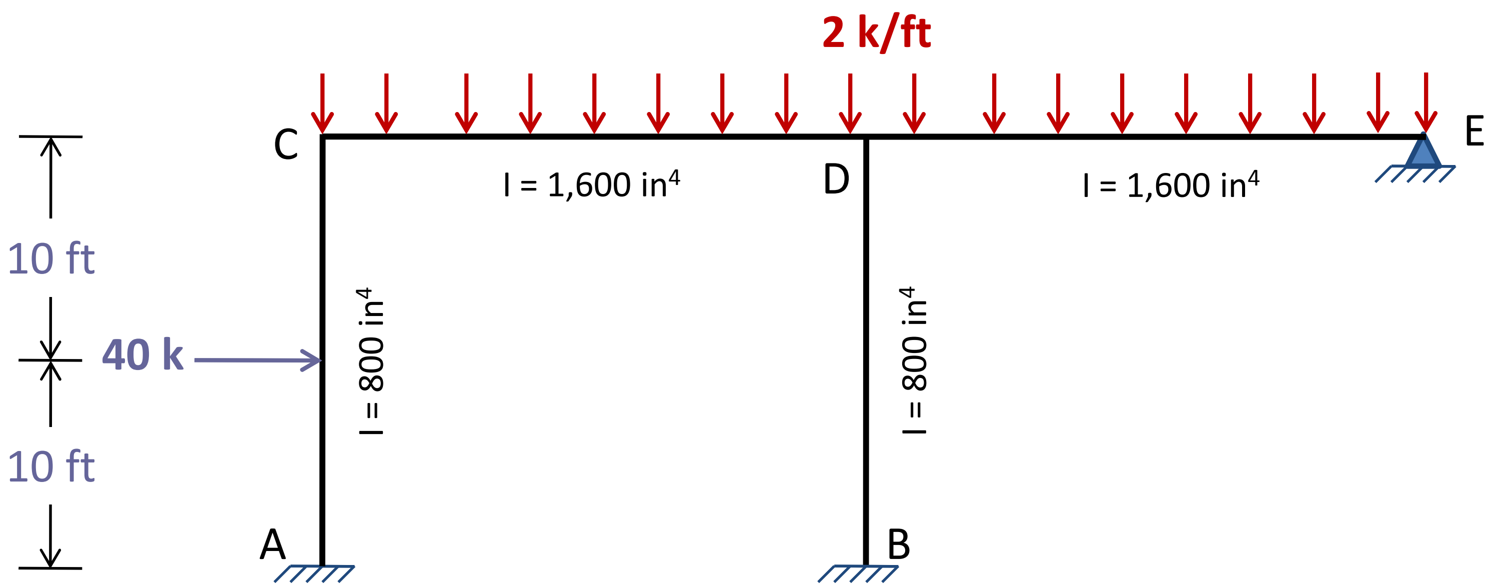
4. Equilibrium Equations

- Apply the moment equilibrium equation, $\sum M = 0$, to the free bodies of the joints C and D.



$$M_{CA} + M_{CD} = 0 \qquad (8)$$

$$M_{DB} + M_{DC} + M_{DE} = 0 \qquad (9)$$



5. Joint Rotations

- Substitution of the SDE into the equilibrium equations yields

$$0.467EI\theta_C + 0.133EI\theta_D = -50 \qquad (10)$$

$$0.133EI\theta_C + 0.667EI\theta_D = -75 \qquad (11)$$

Solve simultaneously,

$$EI\theta_C = -79.545 \ \text{k-ft}^2$$

$$EI\theta_D = -96.591 \ \text{k-ft}^2$$

6. Member End Moments

- The member end moments can now be computed by substituting the values of $EI\theta_C$ and $EI\theta_D$ into the SDE (1 – 7)

$M_{AC} = 92\ \text{k}-\text{ft}$ ANS

$M_{CA} = -115.9\ \text{k}-\text{ft}$ or $115.9\ \text{k-ft}$ ANS

$M_{BD} = -9.7\ \text{k}-\text{ft}$ or $9.7\ \text{k-ft}$ ANS

$M_{DB} = -19.3\ \text{k}-\text{ft}$ or $19.3\ \text{k-ft}$ ANS

$M_{CD} = 115.9\ \text{k}-\text{ft}$ ANS

$M_{DC} = -186.4\ \text{k}-\text{ft}$ or $186.4\ \text{k-ft}$ ANS

$M_{DE} = 205.7\ \text{k}-\text{ft}$ ANS

7. Equilibrium Check

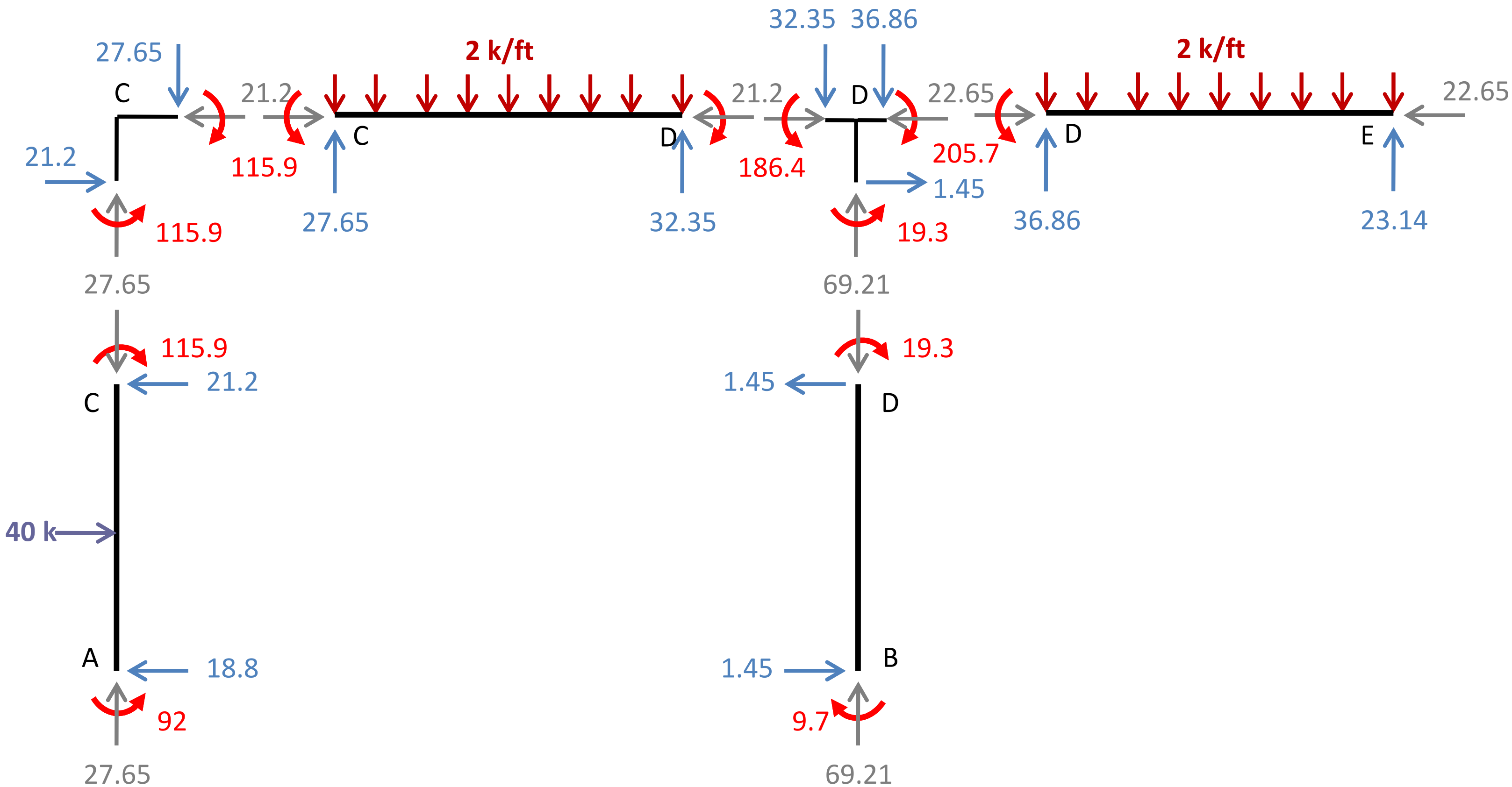
$$M_{CA} + M_{CD} = -115.9 + 115.9 = 0 \qquad \text{Checks}$$

$$M_{DB} + M_{DC} + M_{DE} = -19.3 - 186.4 + 205.7 = 0 \qquad \text{Checks}$$

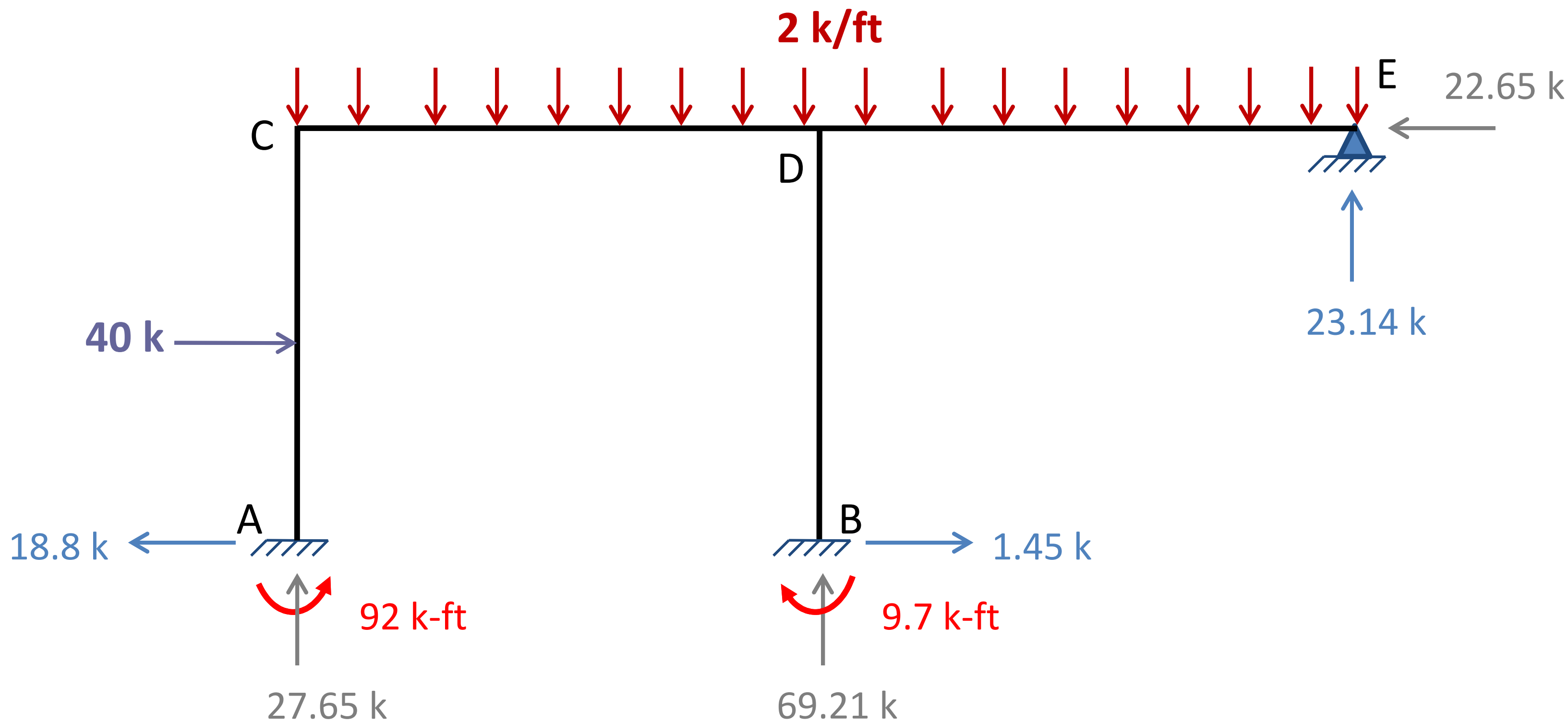
8. Member End Shears

- The member end shears, obtained by considering the equilibrium of each member, are shown on next slide.

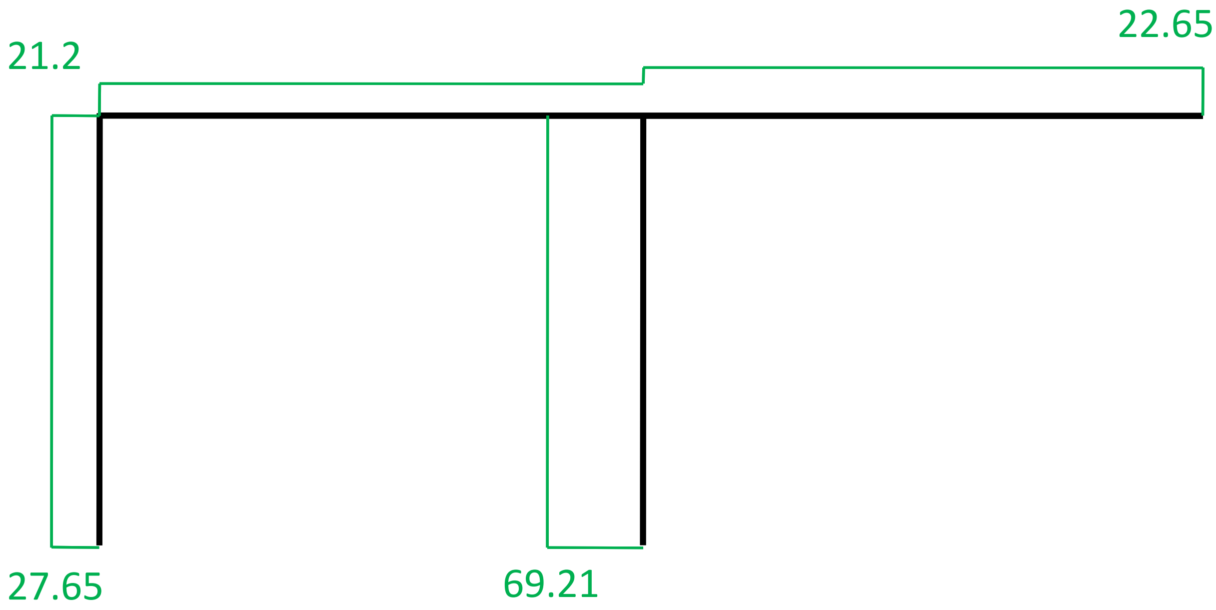
8. Member End Shears and Axial Forces



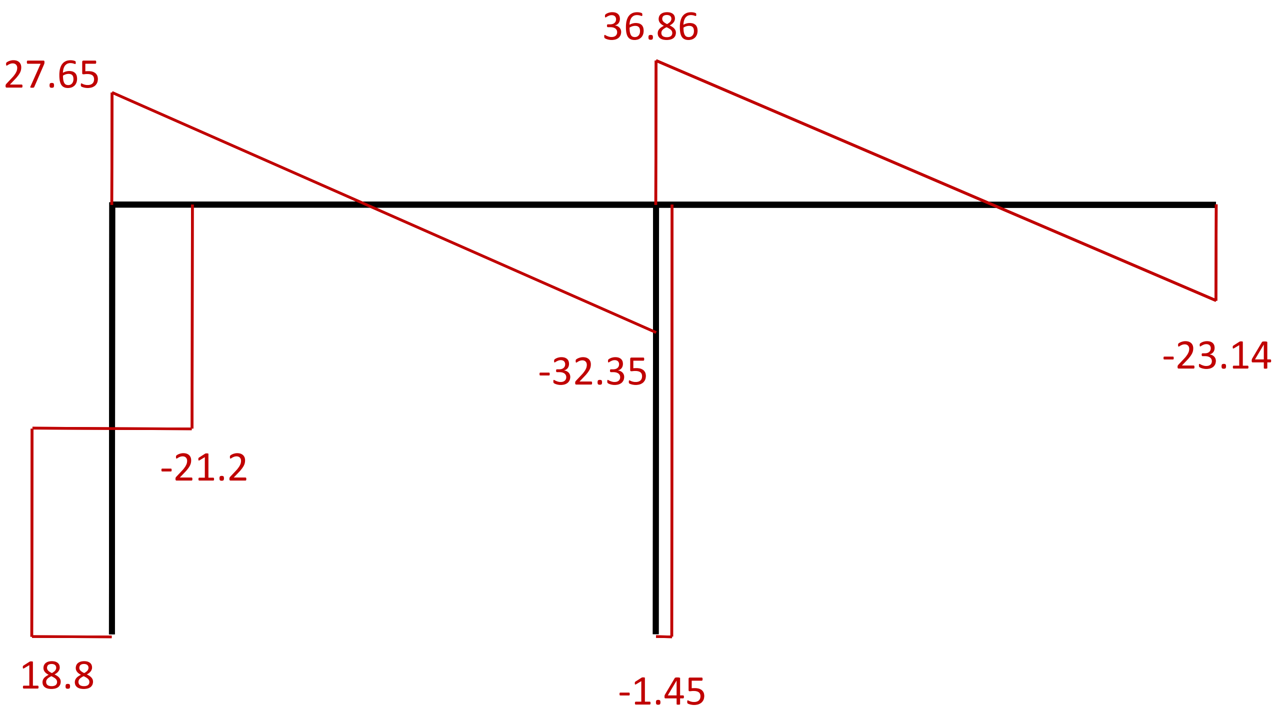
9. Support Reactions



Axial Force Diagram



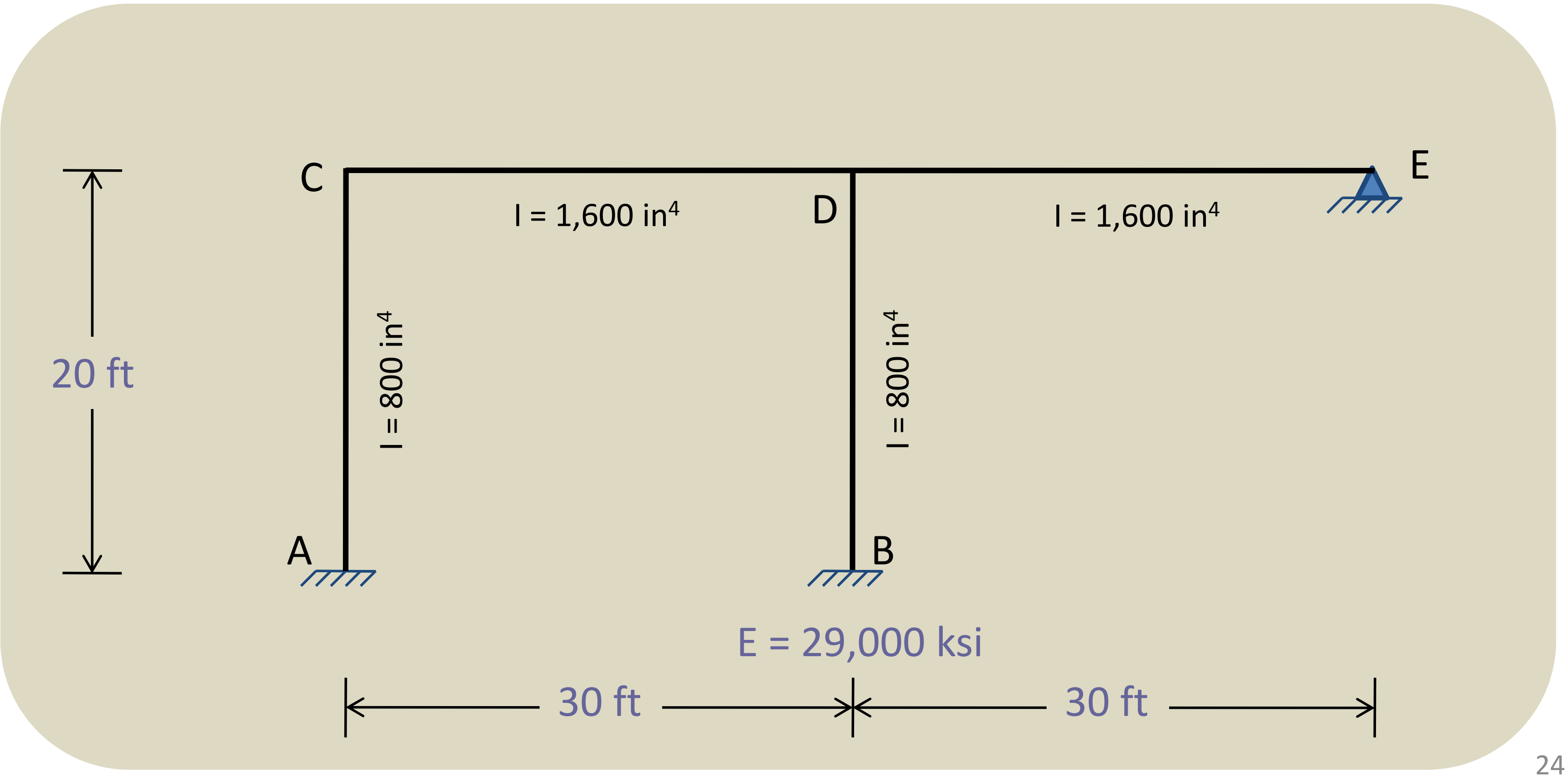
Shear Force Diagram



Bending Moment Diagram

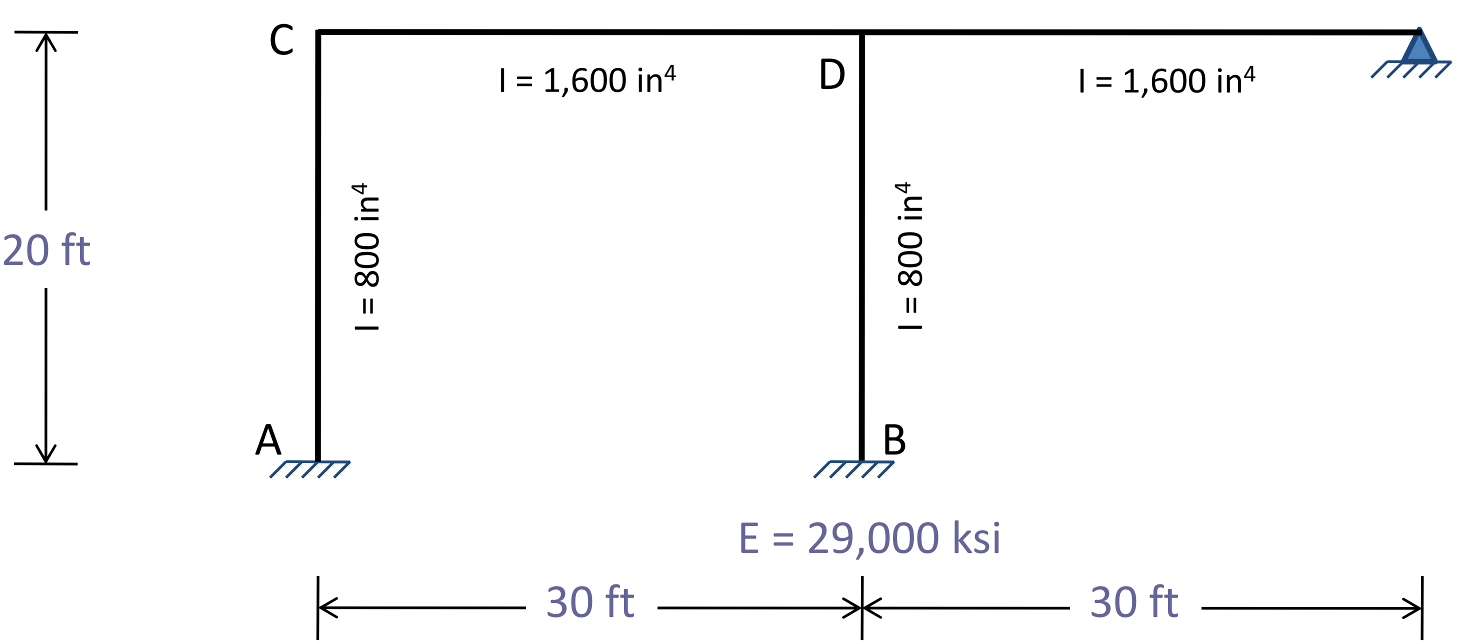
Example 2

- Determine the member end moments and reactions for the frame due to settlement of ¾ in. at support B, shown, by the slope-deflection method.



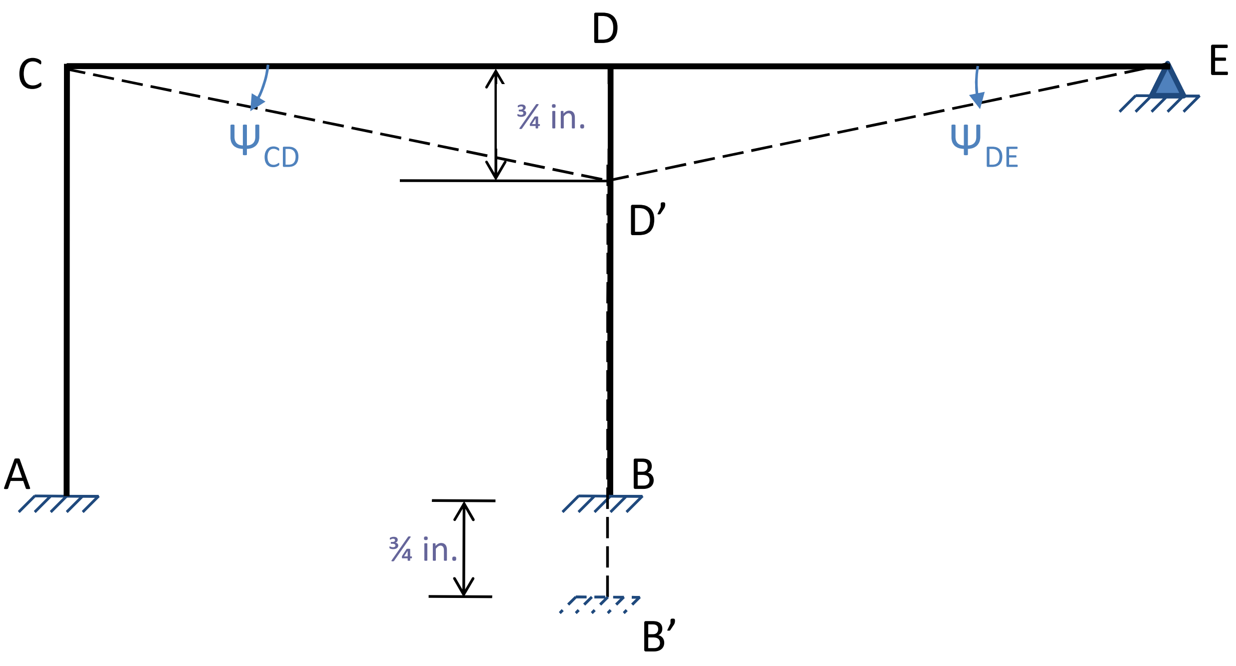
1. Degrees of Freedom

- The analysis involves two unknown joint rotations, θ_C and θ_D.



2. Chord Rotations

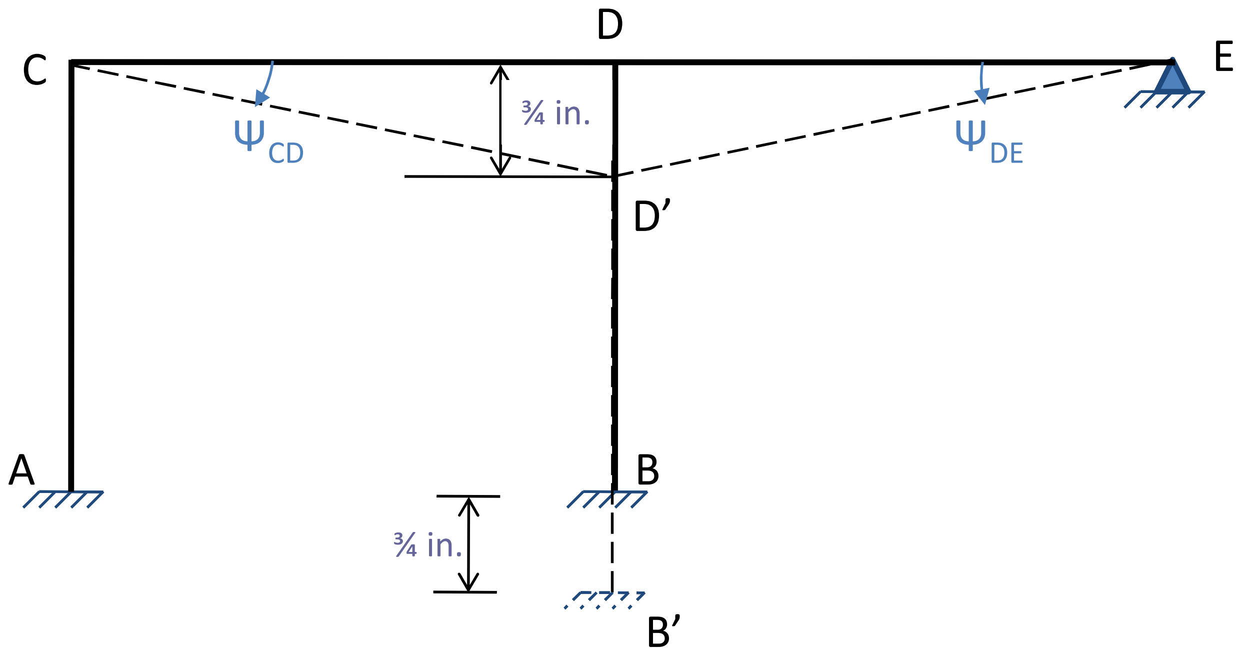
- Since the axial deformation of the member BD is neglected, the ¾ in. settlement of support B causes the joint D to displace downward by the same amount. The inclined dashed lines in this figure represents the chords (not the elastic curve) of members CD and DE in the deformed position.



2. Chord Rotations

$$\psi_{CD} = -\frac{3/4}{(12)(30)} = -0.00208$$

$$\psi_{DE} = \frac{3/4}{(12)(30)} = 0.00208$$



3. Slope-Deflection Equations

$$M_{AC} = 0.1EI\theta_C \qquad (1)$$

$$M_{CA} = 0.2EI\theta_C \qquad (2)$$

$$M_{BD} = 0.1EI\theta_D \qquad (3)$$

$$M_{DB} = 0.2EI\theta_D \qquad (4)$$

$$M_{CD} = \frac{2E(2I)}{30}\left(2\theta_C + \theta_D - 3(-0.00208)\right)$$

$$= 0.267EI\theta_C + 0.133EI\theta_D + 0.000832EI \qquad (5)$$

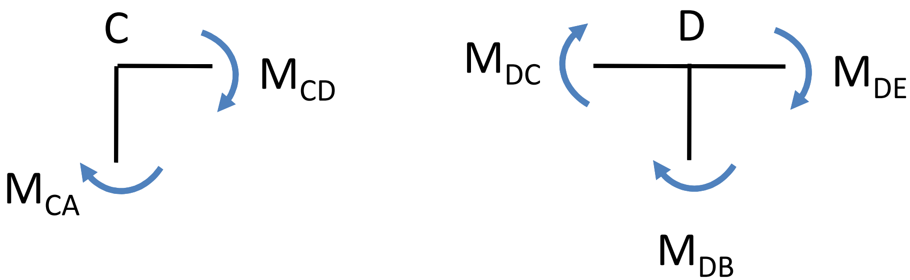
$$M_{DC} = \frac{2E(2I)}{30}\left(2\theta_D + \theta_C - 3(-0.00208)\right)$$

$$= 0.133EI\theta_C + 0.267EI\theta_D + 0.000832EI \qquad (6)$$

$$M_{DE} = \frac{3E(2I)}{30}\left(\theta_D - 0.00208\right) = 0.2EI\theta_D - 0.000416EI \qquad (7)$$

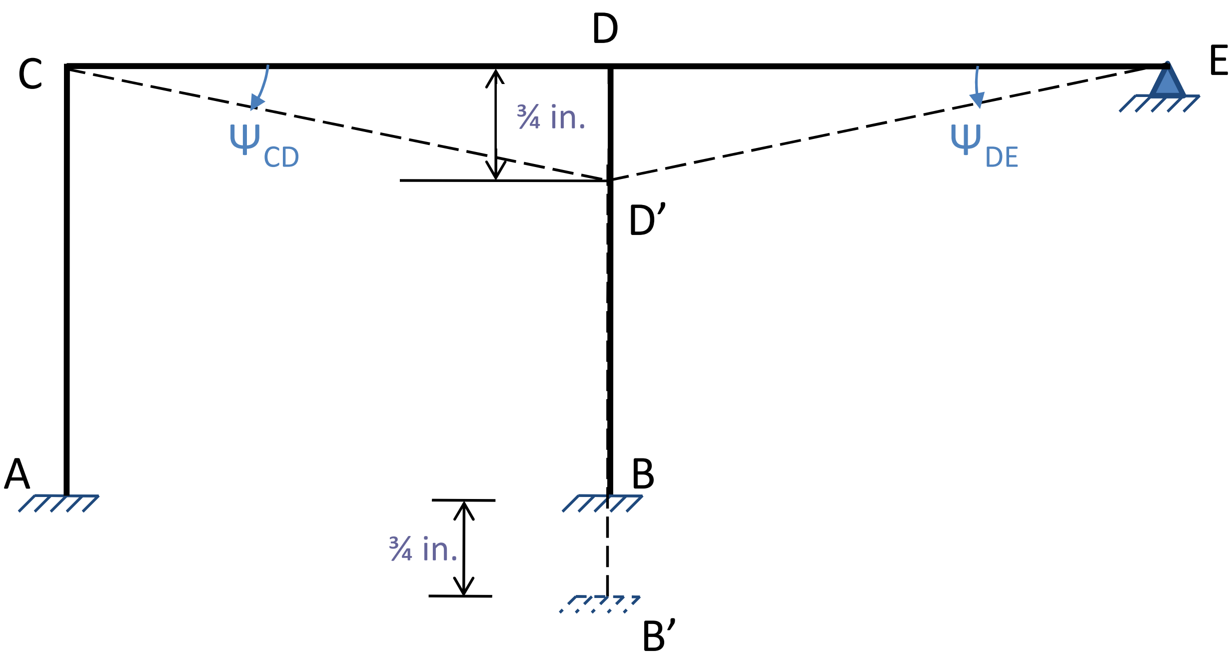
$$M_{ED} = 0 \qquad \text{ANS}$$

4. Equilibrium Equations



$$M_{CA} + M_{CD} = 0 \qquad (8)$$

$$M_{DB} + M_{DC} + M_{DE} = 0 \qquad (9)$$



5. Joint Rotations

- Substitution of the SDE into the equilibrium equations yields

$$0.467EI\theta_C + 0.133EI\theta_D = -0.000832EI$$

$$0.133EI\theta_C + 0.667EI\theta_D = -0.000416EI$$

$$0.467EI\theta_C + 0.133EI\theta_D = -134 \qquad (10)$$

$$0.133EI\theta_C + 0.667EI\theta_D = -67 \qquad (11)$$

Solve simultaneously,

$$EI\theta_C = -273.883 \text{ k-ft}^2$$

$$EI\theta_D = -45.838 \text{ k-ft}^2$$

6. Member End Moments

- The member end moments can now be computed by substituting the values of $EI\theta_C$ and $EI\theta_D$ into the SDE.

$$M_{AC} = -27.4 \text{ k} - \text{ft} \quad \text{or} \quad 27.4 \text{ k-ft} \quad \text{ANS}$$

$$M_{CA} = -54.8 \text{ k} - \text{ft} \quad \text{or} \quad 54.8 \text{ k-ft} \quad \text{ANS}$$

$$M_{BD} = -4.6 \text{ k} - \text{ft} \quad \text{or} \quad 4.6 \text{ k-ft} \quad \text{ANS}$$

$$M_{DB} = -9.2 \text{ k} - \text{ft} \quad \text{or} \quad 9.2 \text{ k-ft} \quad \text{ANS}$$

$$M_{CD} = 54.8 \text{ k} - \text{ft} \quad \text{ANS}$$

$$M_{DC} = 85.4 \text{ k} - \text{ft} \quad \text{ANS}$$

$$M_{DE} = -76.2 \text{ k} - \text{ft} \quad \text{or} \quad 76.2 \text{ k-ft} \quad \text{ANS}$$

7. Equilibrium Check

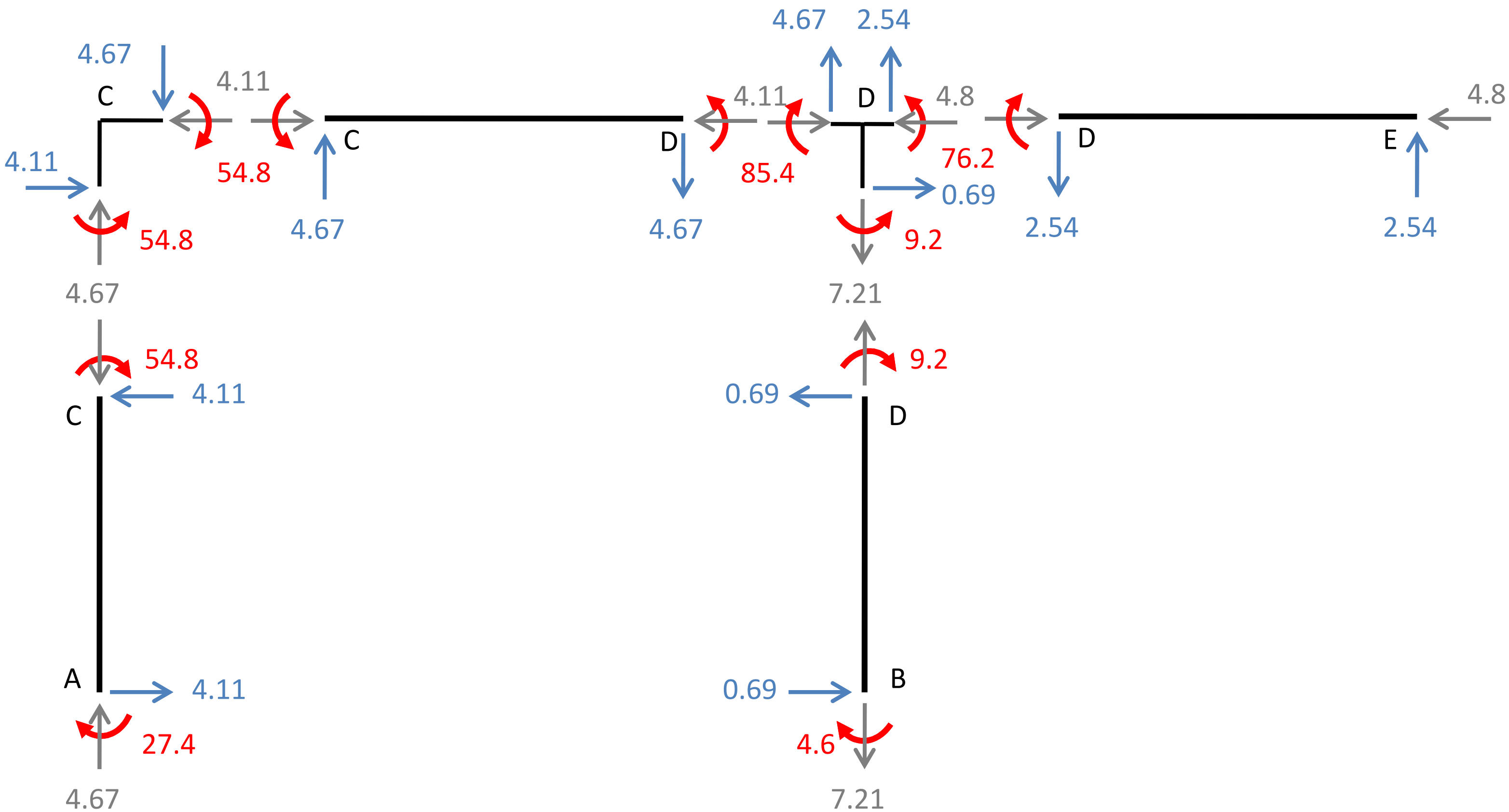
$$M_{CA} + M_{CD} = -54.8 + 54.8 = 0 \qquad \text{Checks}$$

$$M_{DB} + M_{DC} + M_{DE} = -9.2 + 85.4 - 76.2 = 0 \qquad \text{Checks}$$

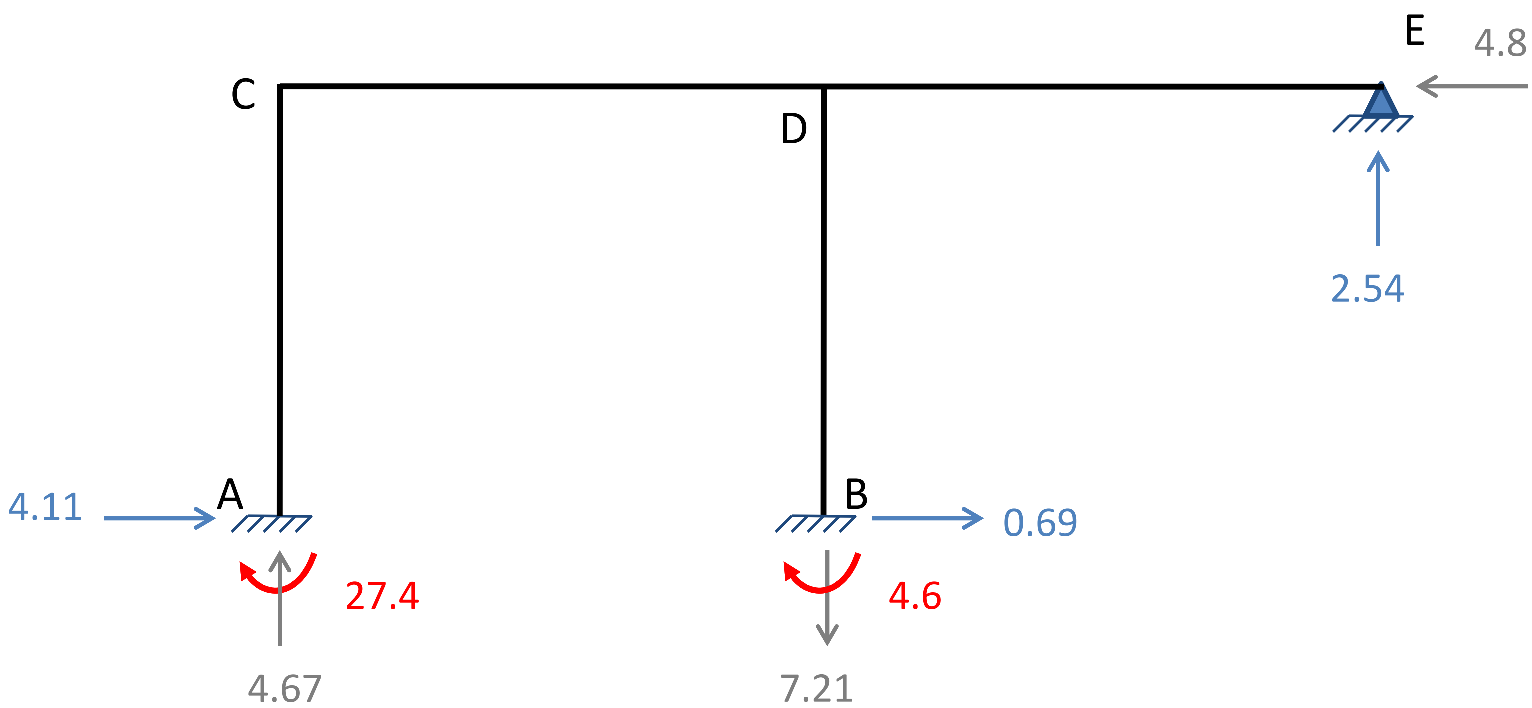
8. Member End Shears

- The member end shears, obtained by considering the equilibrium of each member, are shown on next slide.

8. Member End Shears and Axial Forces

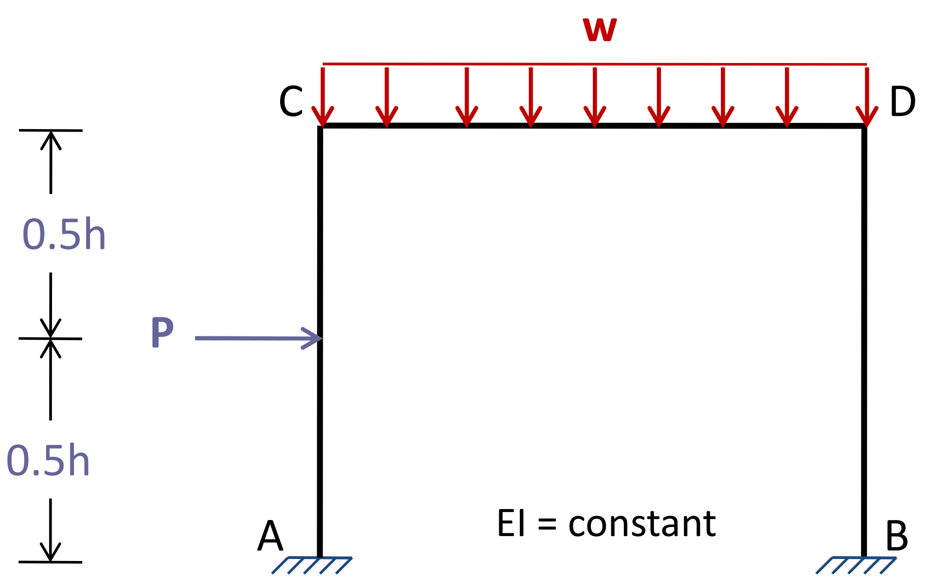


9. Support Reactions

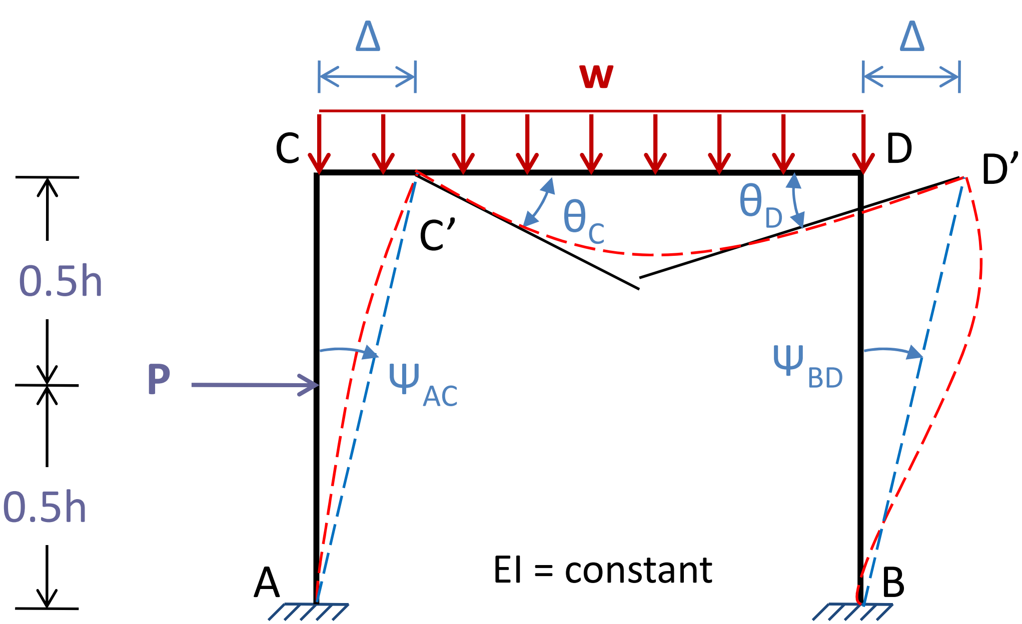


Analysis of Frames With Sidesway

- A frame will undergo sidesway if its joints are not restrained against translation, unless it is a symmetric frame subjected to symmetric loading.
- To develop the analysis of frames with sidesway, consider the rectangular frame shown.



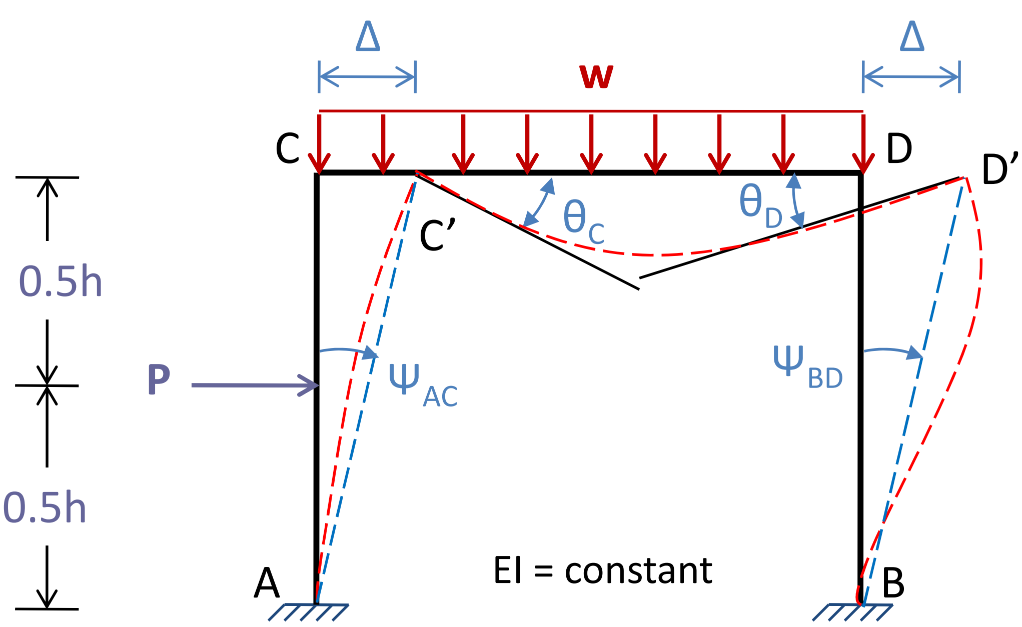
- A qualitative deflected shape of the frame for an arbitrary loading is shown below.
- Fixed joints A and B are completely restrained against rotation and translation.
- The joints C and D are free to rotate and translate.
- Joints C and D can translate only in the horizontal direction as the columns AC and BD are assumed to be inextensible.



Rectangular frame
With sidesway

- The girder CD is also assumed to be inextensible, the horizontal displacement of joints C and D must be the same.

- Thus, the frame has three unknown joint displacements or degrees of freedom, the rotation θ_C and θ_D of joints C and D, respectively, and horizontal displacement Δ of both joints C and D.

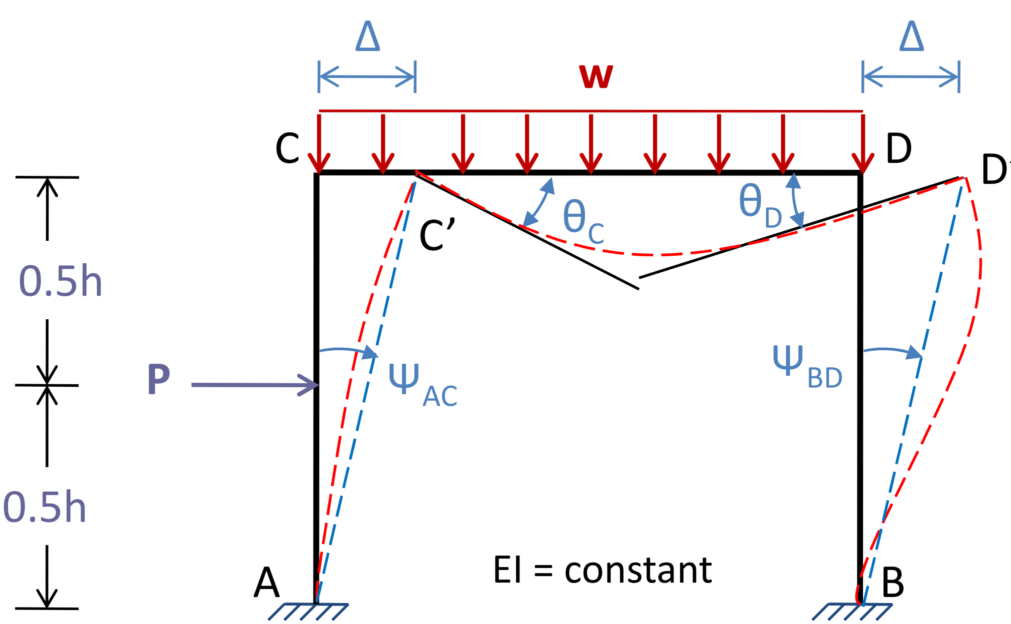


Rectangular frame
With sidesway

- The displacement Δ of joints C and D causes the chords of the columns AC and BD to rotate, and these chord rotations can be expressed in terms of the unknown displacements Δ as

$$\psi_{AC} = \psi_{BD} = -\frac{\Delta}{h} \tag{21}$$

$$\psi_{CD} = 0$$

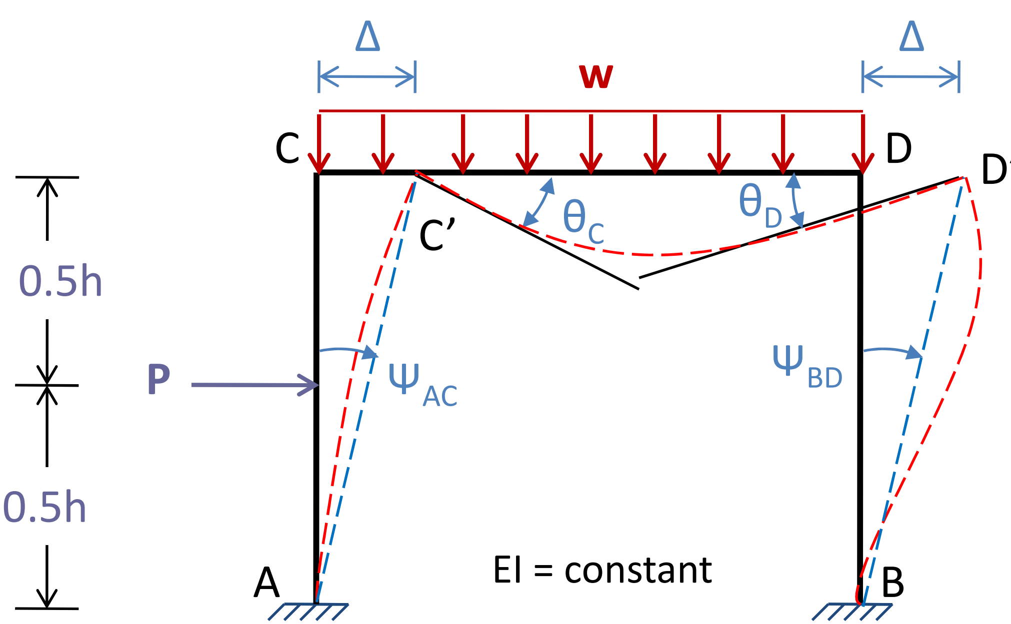


Rectangular frame
With sidesway

- To relate the member end moments to the unknown joint displacements, θ_C, θ_D and Δ we write the slope-deflection equations for the three members of the frame. Applying Eq. 9

$$M_{AC} = \frac{2EI}{h}\left(\theta_C + 3\frac{\Delta}{h}\right) + FEM_{AC} \tag{22a}$$

$$M_{CA} = \frac{2EI}{h}\left(2\theta_C + 3\frac{\Delta}{h}\right) + FEM_{CA} \tag{22b}$$

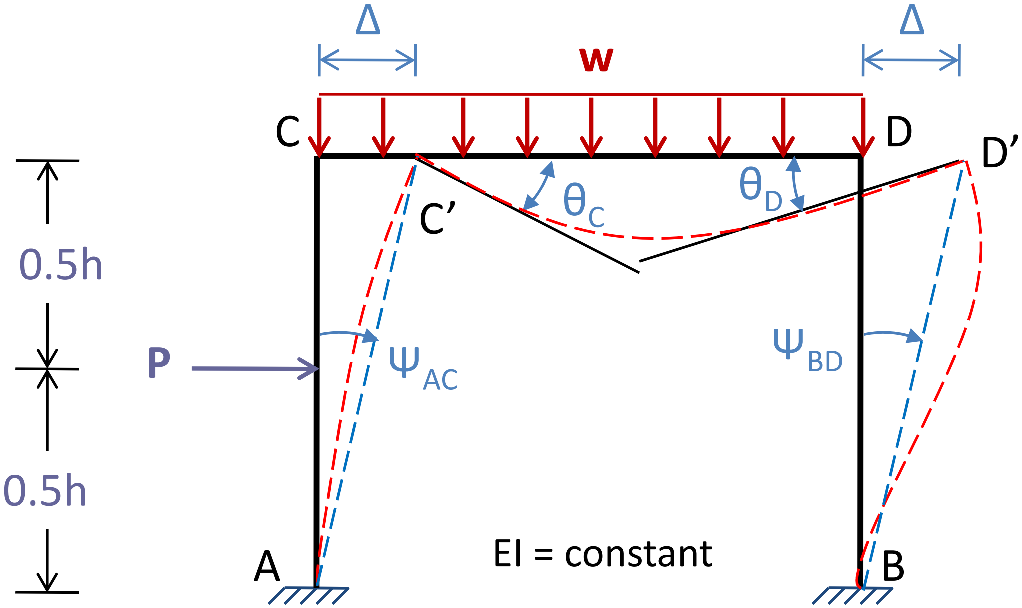


Rectangular frame
With sidesway

- To relate the member end moments to the unknown joint displacements, θ_C, θ_D and Δ we write the slope-deflection equations for the three members of the frame. Applying Eq. 9

$$M_{BD} = \frac{2EI}{h}\left(\theta_D + 3\frac{\Delta}{h}\right) \tag{22c}$$

$$M_{DB} = \frac{2EI}{h}\left(2\theta_D + 3\frac{\Delta}{h}\right) \tag{22d}$$

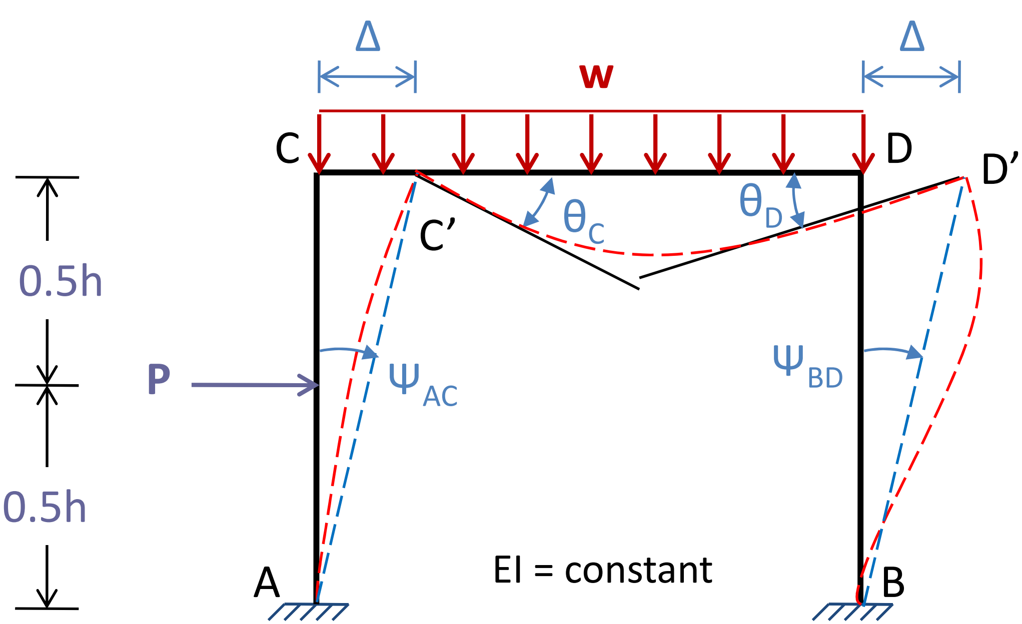


Rectangular frame
With sidesway

- To relate the member end moments to the unknown joint displacements, θ_C, θ_D and Δ we write the slope-deflection equations for the three members of the frame. Applying Eq. 9

$$M_{CD} = \frac{2EI}{L}\left(2\theta_C + \theta_D\right) + FEM_{CD} \tag{22e}$$

$$M_{DC} = \frac{2EI}{L}\left(2\theta_D + \theta_C\right) + FEM_{DC} \tag{22f}$$

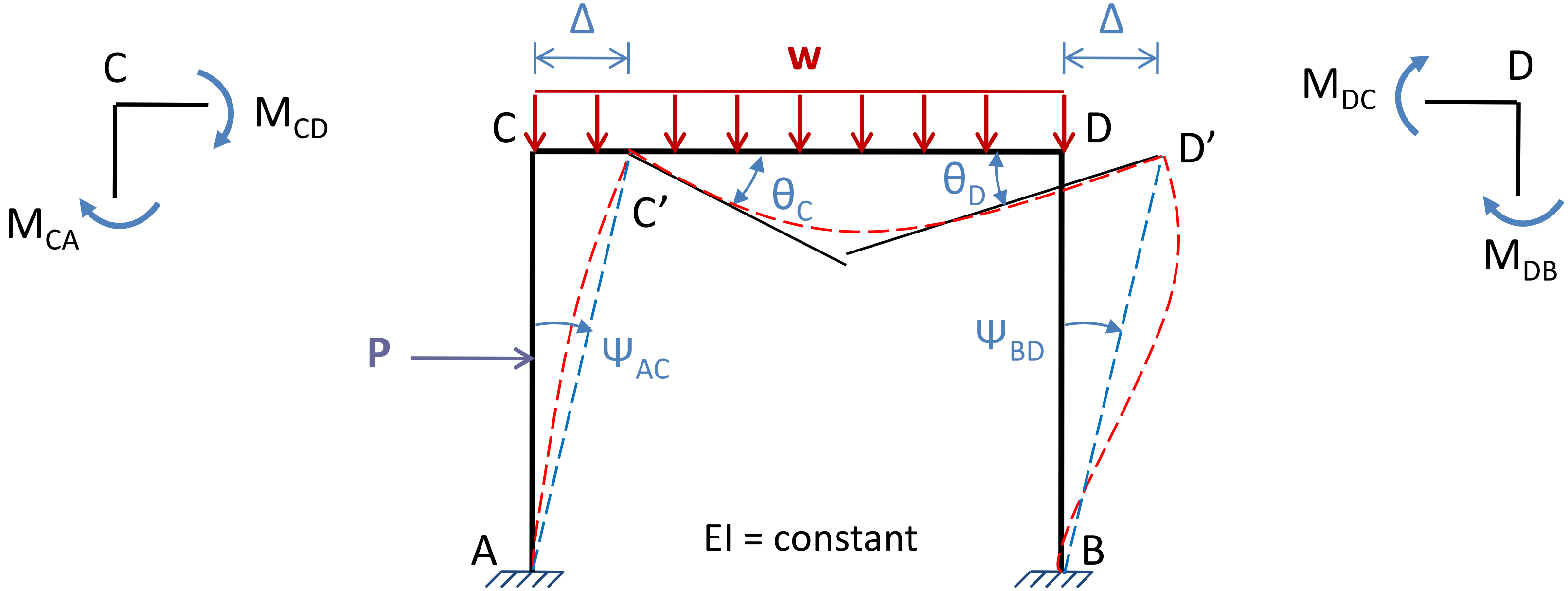


Rectangular frame
With sidesway

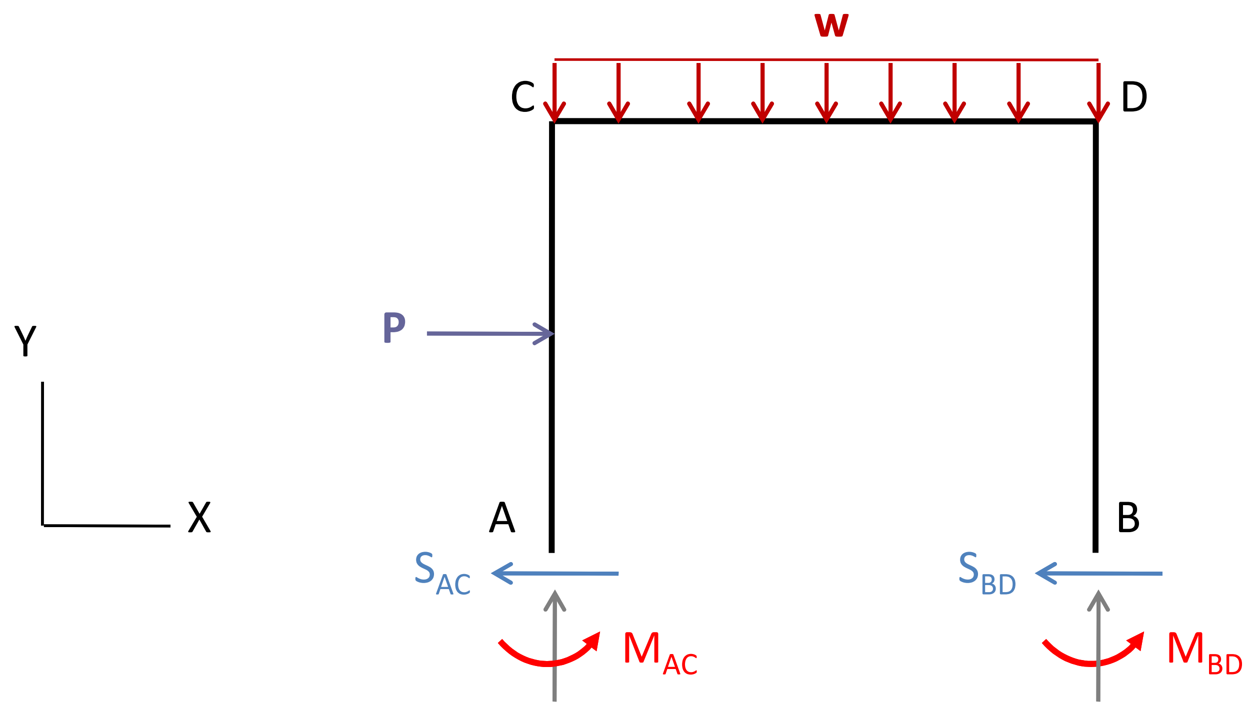
- The three unknowns, θ_C, θ_D and Δ, which must be determined by solving three independent equations of equilibrium before the values of the member end moments can be computed.
- Two of the three equilibrium equations are obtained by considering the moment equilibrium of joints C and D

$$M_{CA} + M_{CD} = 0 \tag{23a}$$

$$M_{DB} + M_{DC} = 0 \tag{23b}$$



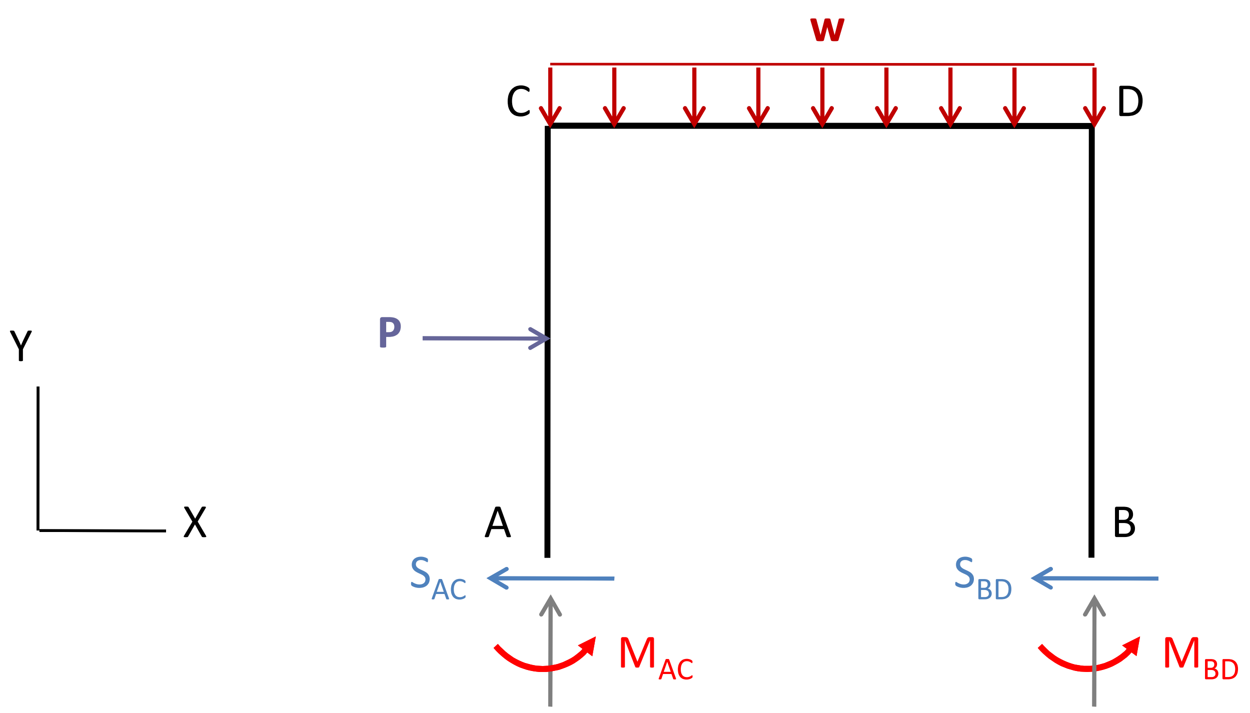
- The third equilibrium equation, commonly termed the SHEAR EQUATION is based on the condition that the sum of all the horizontal forces acting on the free body of the entire frame must be zero.
- The free body diagram of the frame, obtained by passing an imaginary section just above the support level is shown in figure below.



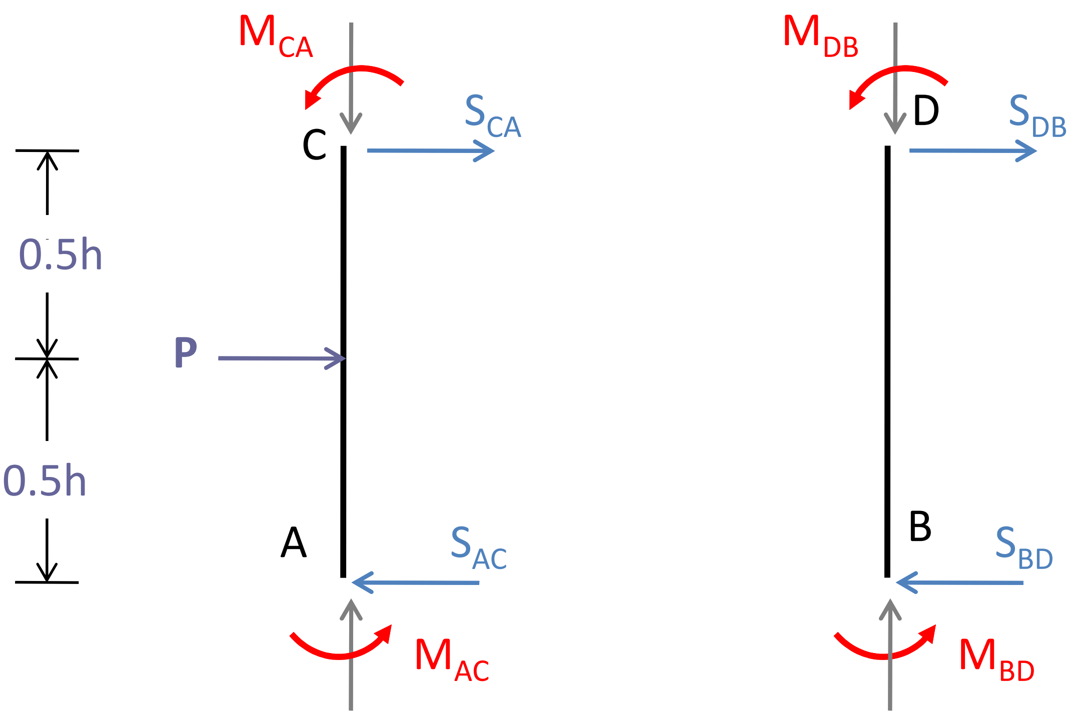
- By applying the equilibrium equation $\sum F_X = 0$, we write

$$P - S_{AC} - S_{BD} = 0 \tag{23c}$$

in which S_{AC} and S_{BD} are the shears at lower ends of the columns AC and BD, respectively.



- To express the third equilibrium equation in terms of column end moments, we consider the equilibrium of the free bodies of the columns AC and BD as shown.

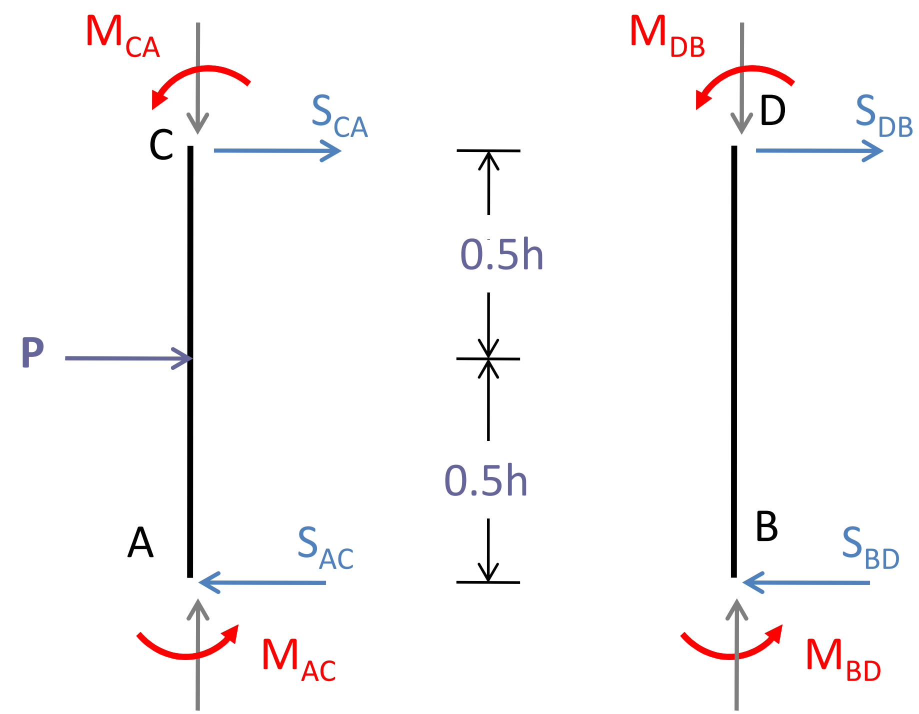


- By summing moments about the top of each column, we obtain the following

$$+ \quad \sum M_C^{AC} = 0$$

$$M_{AC} - S_{AC}(h) + P\left(\frac{h}{2}\right) + M_{CA} = 0$$

$$S_{AC} = \frac{M_{AC} + M_{CA}}{h} + \frac{P}{2} \qquad (24a)$$



$$+ \quad \sum M_D^{BD} = 0$$

$$M_{BD} + M_{DB} - S_{DB}(h) = 0$$

$$S_{BD} = \frac{M_{BD} + M_{DB}}{h} \qquad (24b)$$

- By substituting Eqs. 24a and 24b into Eq. 23c, we obtain the third equilibrium equation in terms of member end moments

$$P - S_{AC} - S_{BD} = 0 \tag{23c}$$

$$P - \left(\frac{M_{AC} + M_{CA}}{h} + \frac{P}{2} \right) - \left(\frac{M_{BD} + M_{DB}}{h} \right) = 0$$

which reduces to

$$M_{AC} + M_{CA} + M_{BD} + M_{DB} - \frac{Ph}{2} = 0 \tag{25}$$

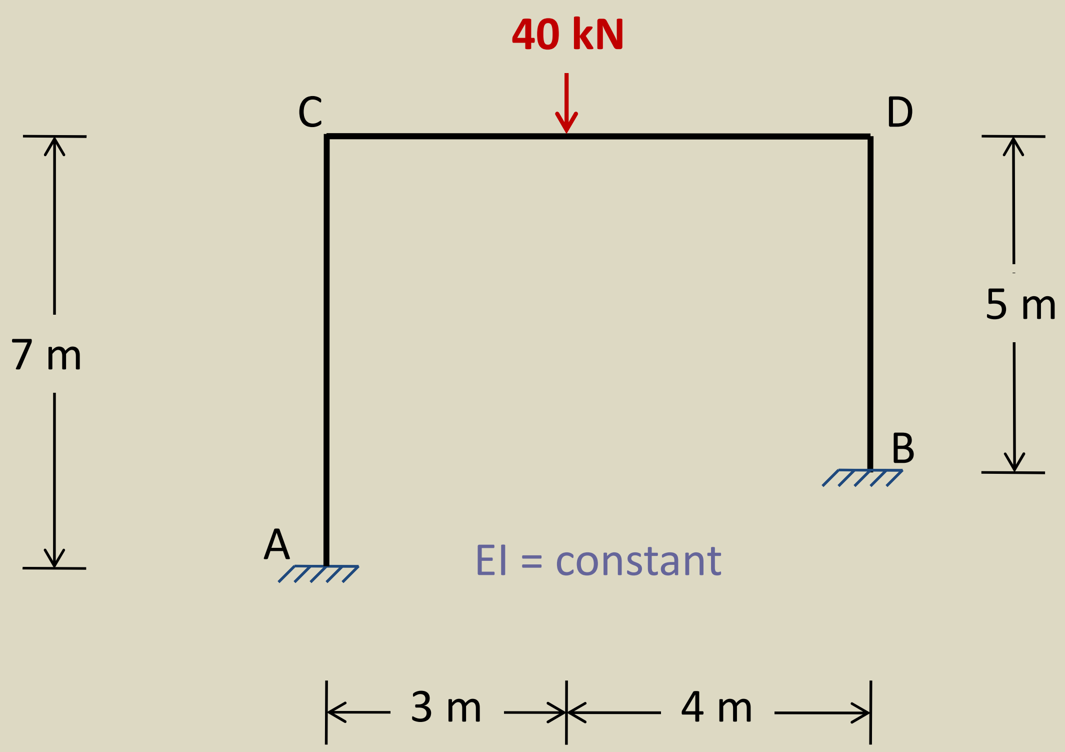
- With the three equilibrium equations (Eqs. 23a & 23b and 25 now established, we can proceed with the rest of analysis in the ususal manner.

- By substituting the slope-deflection equations (Eqs. 22) into the equilibrium equations, we obtain the systems of equations that can be solved for the unknown joint displacements θ_C, θ_D and Δ.

- The joint displacements thus obtained then can be back submitted into the slope-deflection equations to determine the member end moments, from which end shears and axial forces of members and the supports reactions can be computed, as discuss previously.

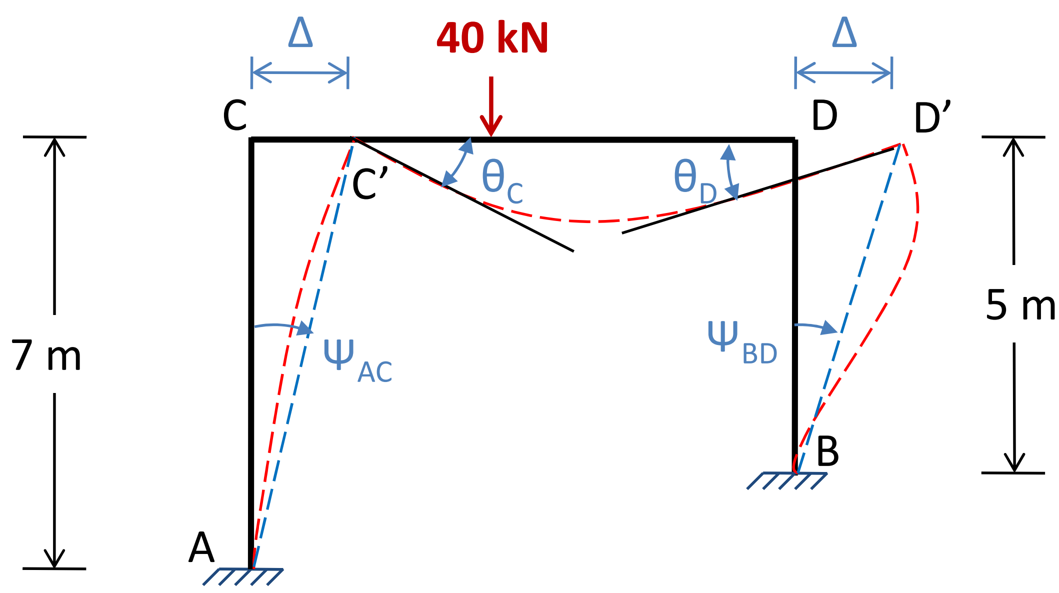
Example 3

- Determine the member end moments and reactions for the frame shown by the slope-deflection method.



1. Degrees of Freedom

- The degrees of freedom are θ_C, θ_D. and Δ.



2. Fixed-End Moments

- By using the fixed end moments expressions given inside the back cover of the book, we obtain

$$FEM_{CD} = \frac{40(3)(4)^2}{(7)^2} = 39.2 \text{ kN.m} \qquad \text{or} \qquad +39.2 \text{ kN.m}$$

$$FEM_{DC} = \frac{40(3)^2(4)}{(7)^2} = 29.4 \text{ kN.m} \qquad \text{or} \qquad -29.4 \text{ kN.m}$$

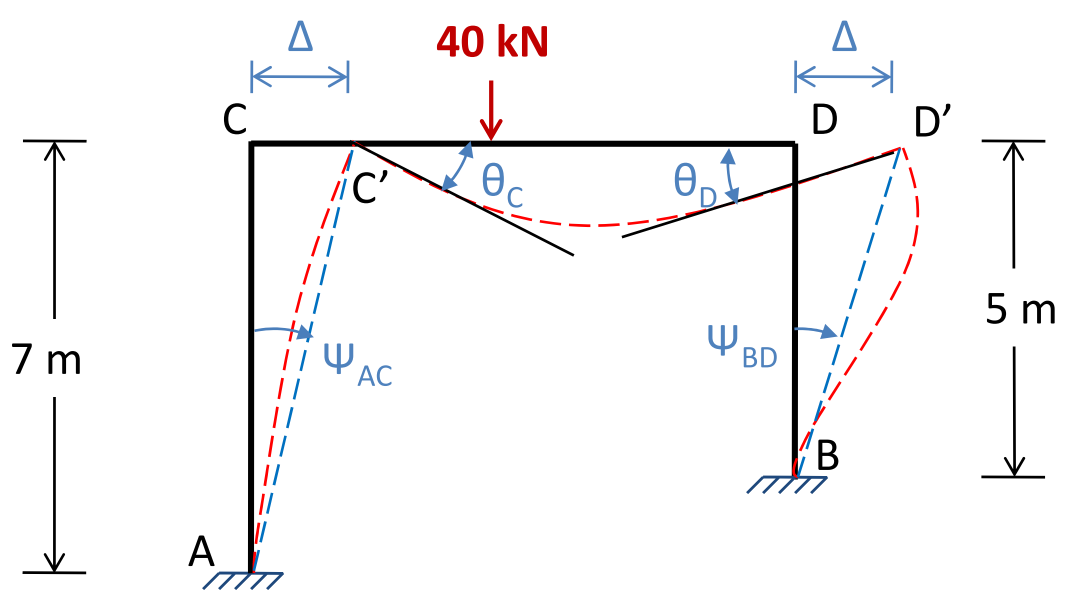
$$FEM_{CD} = FEM_{CA} = FEM_{BD} = FEM_{DB} = 0$$

3. Chord Rotations

$$\psi_{AC} = -\frac{\Delta}{7}$$

$$\psi_{BD} = -\frac{\Delta}{5}$$

$$\psi_{CD} = 0$$



4. Slope-Deflection Equations

$$M_{AC} = \frac{2EI}{7}\left(\theta_C - 3\left(-\frac{\Delta}{7}\right)\right) = 0.286EI\theta_C + 0.122EI\Delta \tag{1}$$

$$M_{CA} = \frac{2EI}{7}\left(2\theta_C - 3\left(-\frac{\Delta}{7}\right)\right) = 0.571EI\theta_C + 0.122EI\Delta \tag{2}$$

$$M_{BD} = \frac{2EI}{5}\left(\theta_D - 3\left(-\frac{\Delta}{5}\right)\right) = 0.4EI\theta_D + 0.24EI\Delta \tag{3}$$

$$M_{DB} = \frac{2EI}{5}\left(2\theta_D - 3\left(-\frac{\Delta}{5}\right)\right) = 0.8EI\theta_D + 0.24EI\Delta \tag{4}$$

$$M_{CD} = \frac{2EI}{7}\left(2\theta_C + \theta_D\right) + 39.2 = 0.571EI\theta_C + 0.286EI\theta_D + 39.2 \tag{5}$$

$$M_{DC} = \frac{2EI}{7}\left(\theta_C + 2\theta_D\right) - 29.4 = 0.286EI\theta_C + 0.571EI\theta_D - 29.4 \tag{6}$$

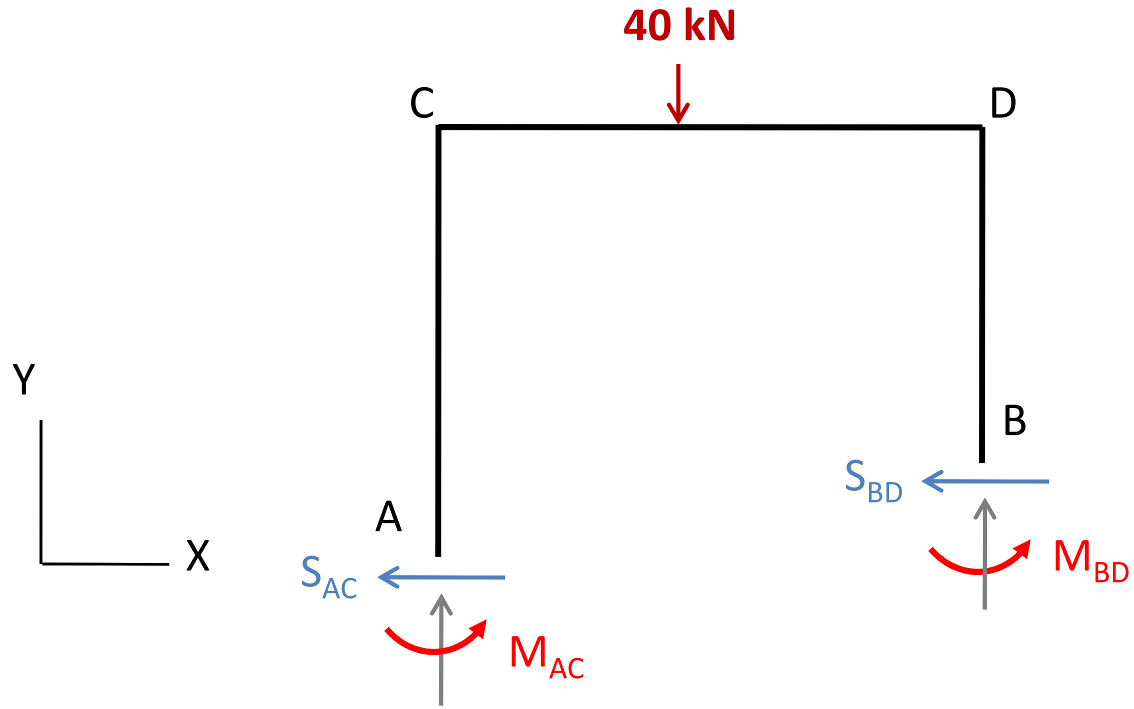
5. Equilibrium Equations

$$M_{CA} + M_{CD} = 0 \quad (7)$$

$$M_{DB} + M_{DC} = 0 \quad (8)$$

To establish the third equilibrium equation, we apply the force equilibrium equation $\sum F_X = 0$ to the free body of the entire frame, to obtain

$$S_{AC} + S_{BD} = 0$$



5. Equilibrium Equations

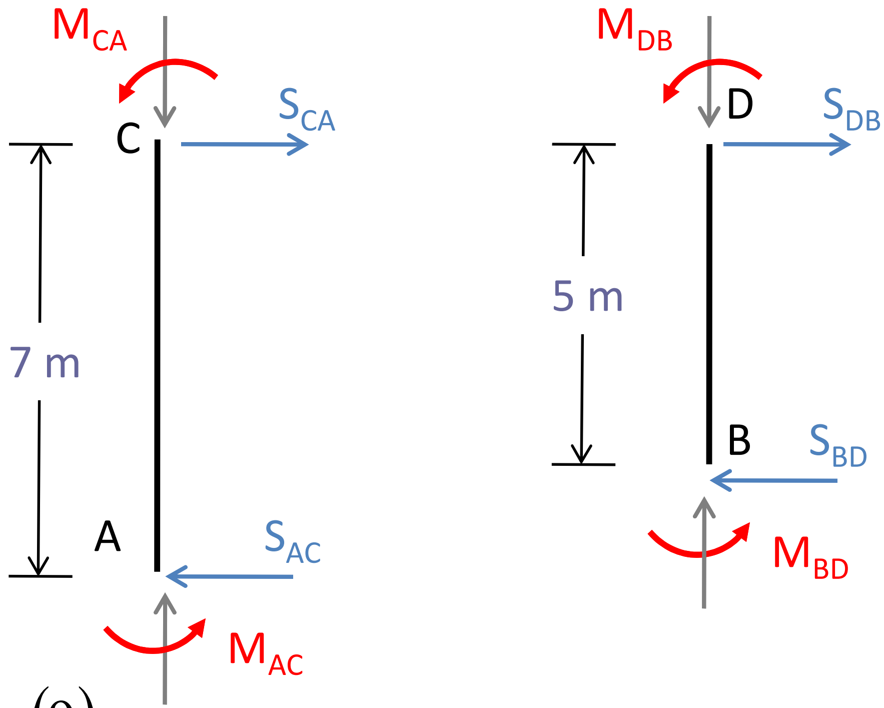
To express the column end shears in terms of column end moments, we draw the free body diagram of the two columns and sum the moments about the top of each column:

$$S_{AC} + S_{BD} = 0$$

$$S_{AC} = \frac{M_{AC} + M_{CA}}{7}$$

$$S_{BD} = \frac{M_{BD} + M_{DB}}{5}$$

$$\frac{M_{AC} + M_{CA}}{7} + \frac{M_{BD} + M_{DB}}{5} = 0$$



$$5(M_{AC} + M_{CA}) + 7(M_{BD} + M_{DB}) = 0 \qquad (9)$$

6. Joints Displacements

To determine the unknown joint displacements θ_C, θ_D and Δ, we substitute the SDE (1 to 6) into the equilibrium equations (7 to 9) to obtain

$$1.142EI\theta_C + 0.286EI\theta_D + 0.122EI\Delta = -39.2 \qquad (10)$$

$$0.286EI\theta_C + 1.371EI\theta_D + 0.24EI\Delta = 29.4 \qquad (11)$$

$$4.285EI\theta_C + 8.4EI\theta_D + 4.58EI\Delta = 0 \qquad (12)$$

Solving Eqs. 10 to 12 simultaneously yields

$$EI\theta_C = -40.211 \text{ kN.m}^2$$

$$EI\theta_D = 34.24 \text{ kN.m}^2$$

$$EI\Delta = -25.177 \text{ kN.m}^3$$

7. Member End Moments

By substituting the numerical values of $EI\theta_C$, $EI\theta_D$, and $EI\Delta$ into the slope-deflection equations (Eqs. 1 through 6), we obtain

$M_{AC} = -14.6\ \text{kN.m}$ or $14.6\ \text{kN.m}$ ANS

$M_{CA} = -26\ \text{kN.m}$ or $26\ \text{kN.m}$ ANS

$M_{BD} = 7.7\ \text{kN.m}$ ANS

$M_{DB} = 21.3\ \text{kN.m}$ ANS

$M_{CD} = 26\ \text{kN.m}$ ANS

$M_{DC} = -21.3\ \text{kN.m}$ or $21.3\ \text{kN.m}$ ANS

8. Equilibrium Check

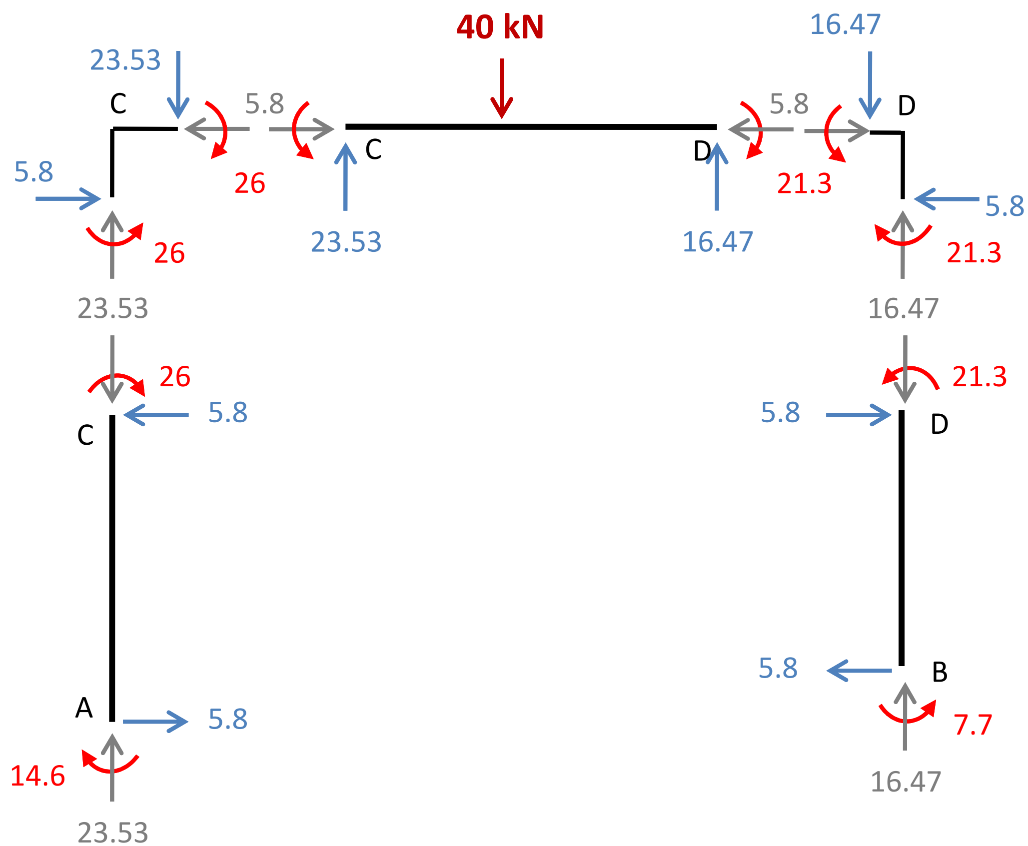
By substituting the numerical values of $EI\theta_C$, $EI\theta_D$, and $EI\Delta$ into the slope-deflection equations (Eqs. 1 through 6), we obtain

$$M_{CA} + M_{CD} = -26 + 26 = 0 \qquad \text{Checks}$$

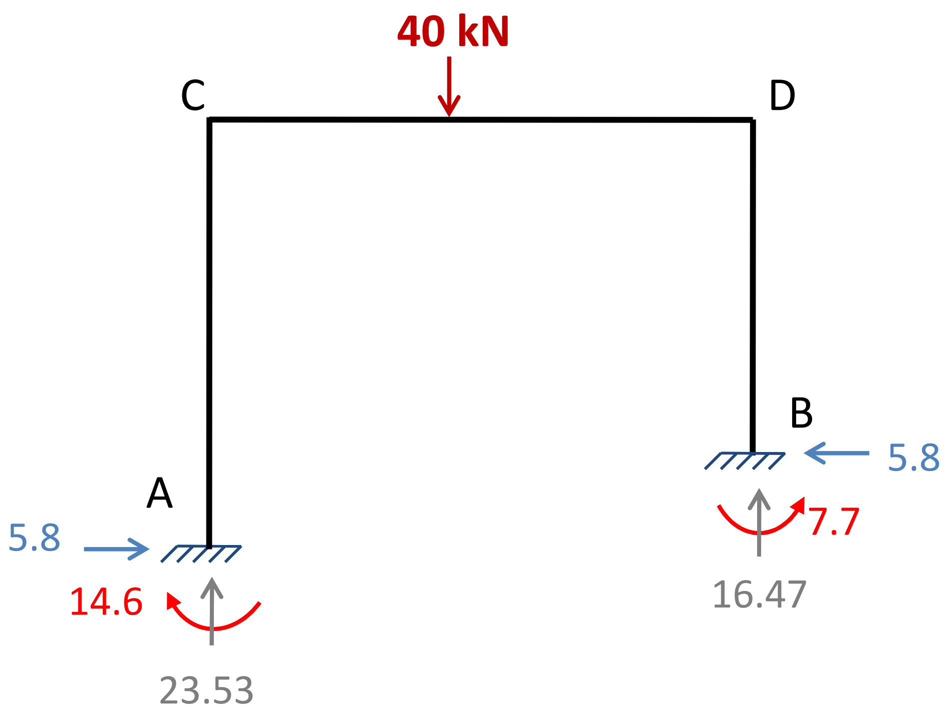
$$M_{DB} + M_{DC} = 21.3 - 21.3 = 0 \qquad \text{Checks}$$

$$5(M_{AC} + M_{CA}) + 7(M_{BD} + M_{DB}) = 5(-14.6 - 26) + 7(7.7 + 21.3) = 0 \qquad \text{Checks}$$

9. Member End Shears and Axial Forces



10. Support Reactions



Frames with Inclined Legs

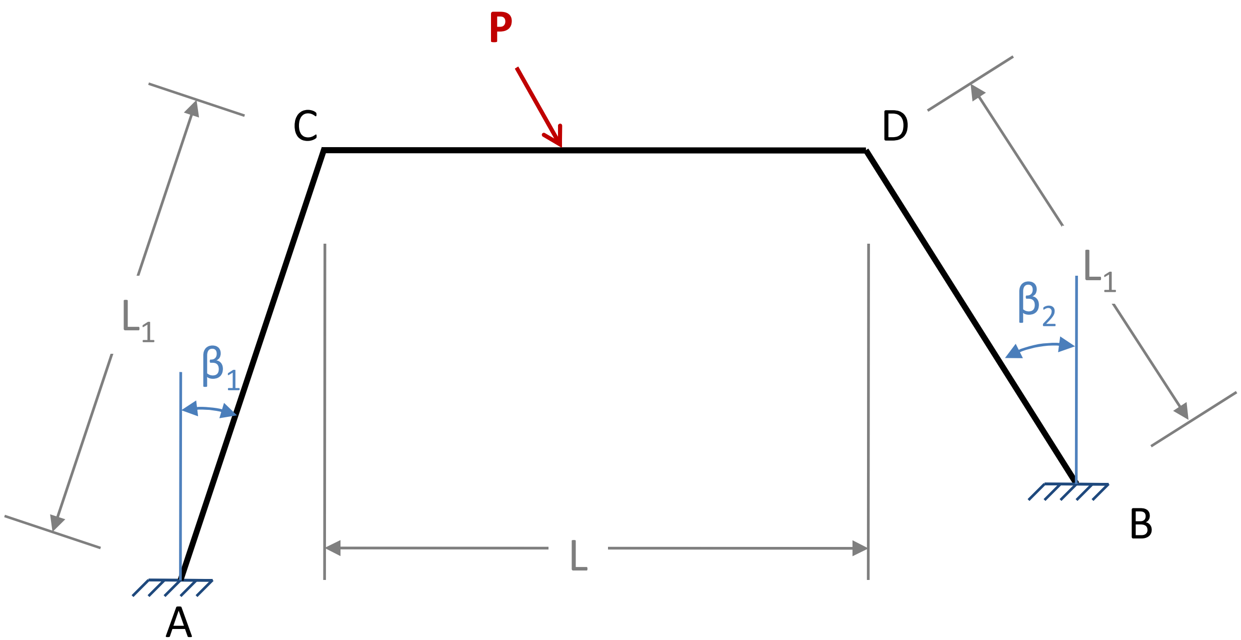
- The analysis of frames with inclined legs is similar to that of the rectangular frames considered previously.

- But when frames with inclined legs are subjected to sidesway, their horizontal members also undergo chord rotations, which must be included in the analysis.

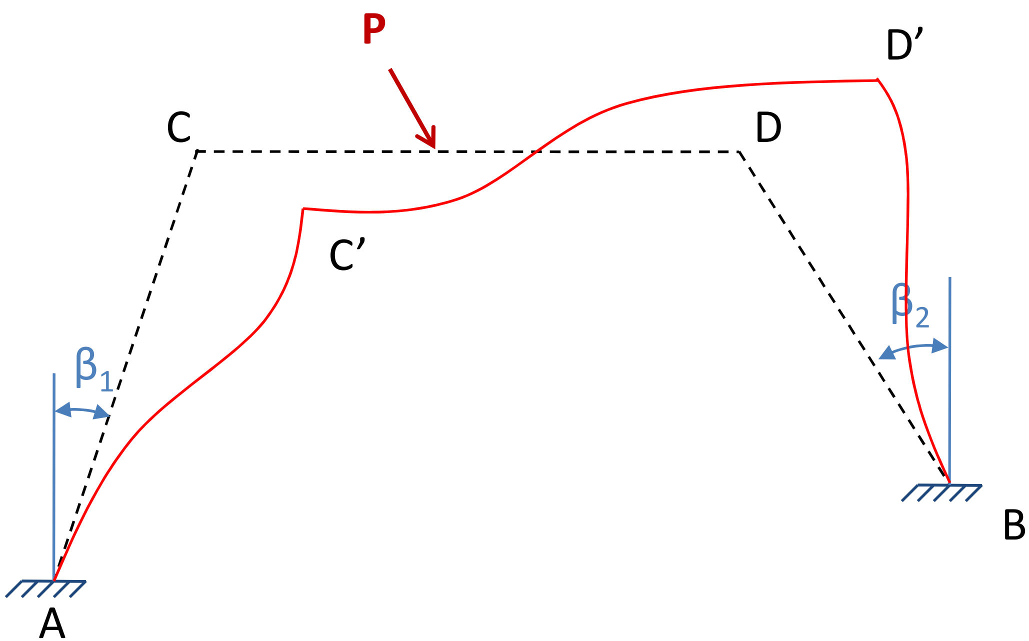
- Recall that the chord rotations of the horizontal members of rectangular frames, subjected to sideway, are zero.

- Consider the frame with inclined legs shown in figure below.

- In order to analyze this frame by the slope-deflection method, we must relate the chord rotations of its three members to each other or to an independent joint translation.

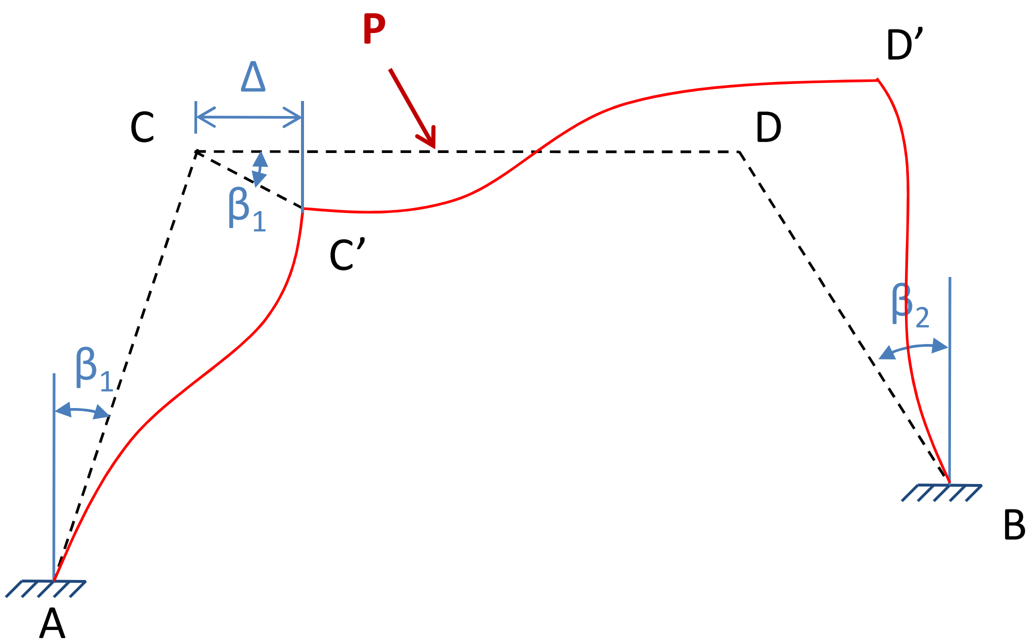


- We subject the joint C of the frame to an arbitrary horizontal displacement Δ and draw a qualitative deflected shape of the frame.
- This is consistent with its supports conditions as well as with our assumptions that the members of the frame are in-extensible.
- The deflected shape is shown below.



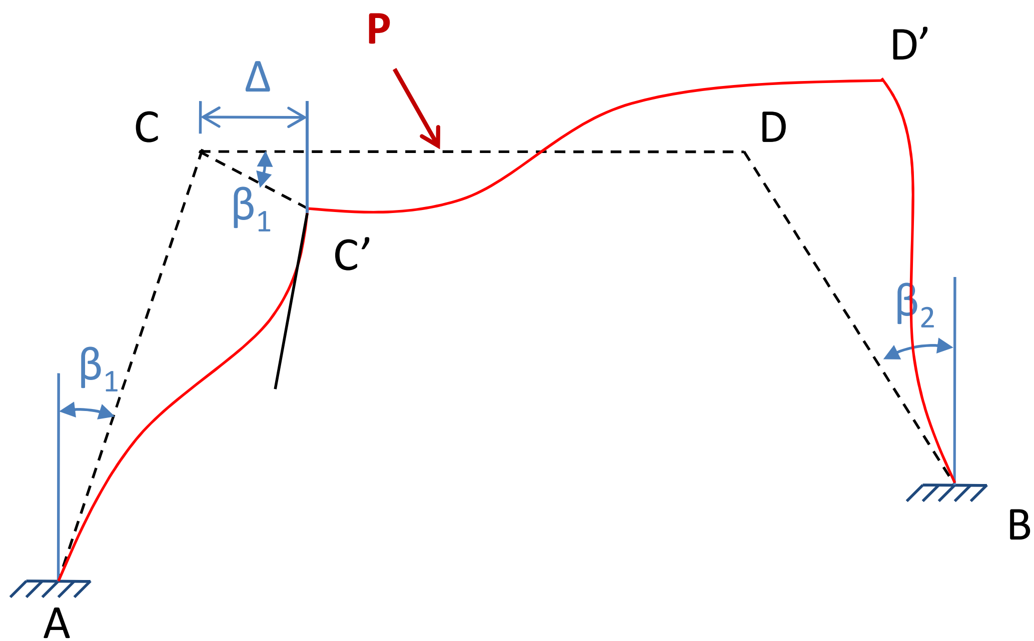
Deflected shape of the frame due to sidesway

- First imagine that the members BD and CD are disconnected at joint D.
- Since member AC is assumed to be in-extensible, joint C can move only in an arc about point A.
- Translation of joint C is assumed to be small, we can consider the arc to be a straight line perpendicular to member AC.



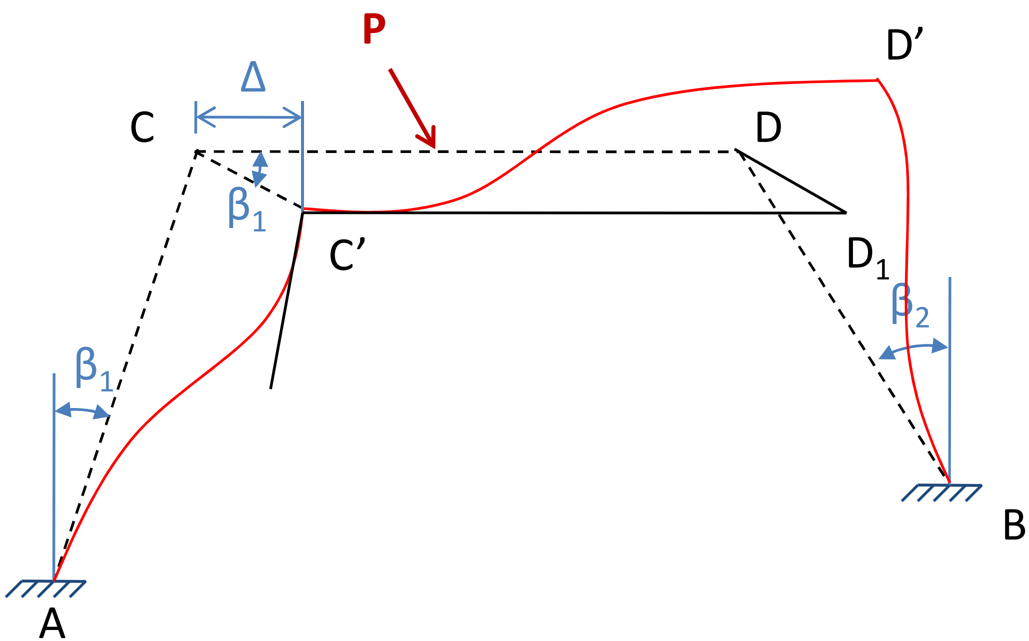
Deflected shape of the frame due to sidesway

- In order to move joint C horizontally by a distance Δ, we must displace it in a direction perpendicular to member AC by a distance CC’, so that the horizontal component of CC’ equals Δ.
- Although, joint C is free to rotate its rotation is ignored at this stage of the analysis and the elastic curve AC’ of member AC is drawn with the tangent at C’ parallel to the undeformed direction of the member.



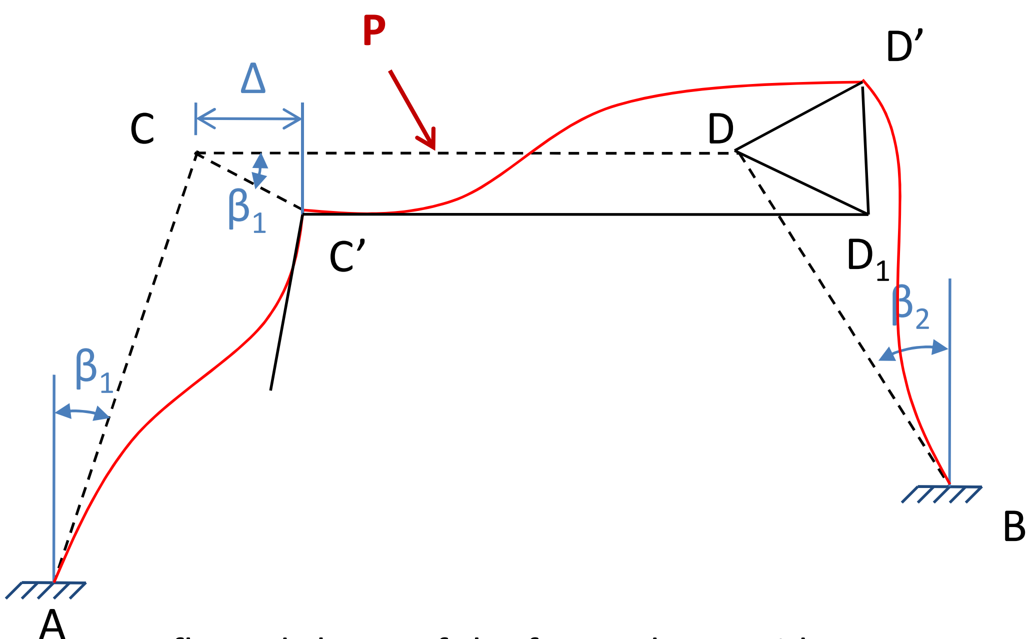
Deflected shape of the frame due to sidesway

- The member CD remains horizontal and translates as a rigid body into the position $C'D_1$ with the displacement DD_1 equal to CC' as shown.
- Since horizontal member CD is assumed to be inextensible and the translation of joint D is assumed to be small, the end D of this member can be moved from its deformed position D_1 only in the vertical direction.



Deflected shape of the frame due to sidesway

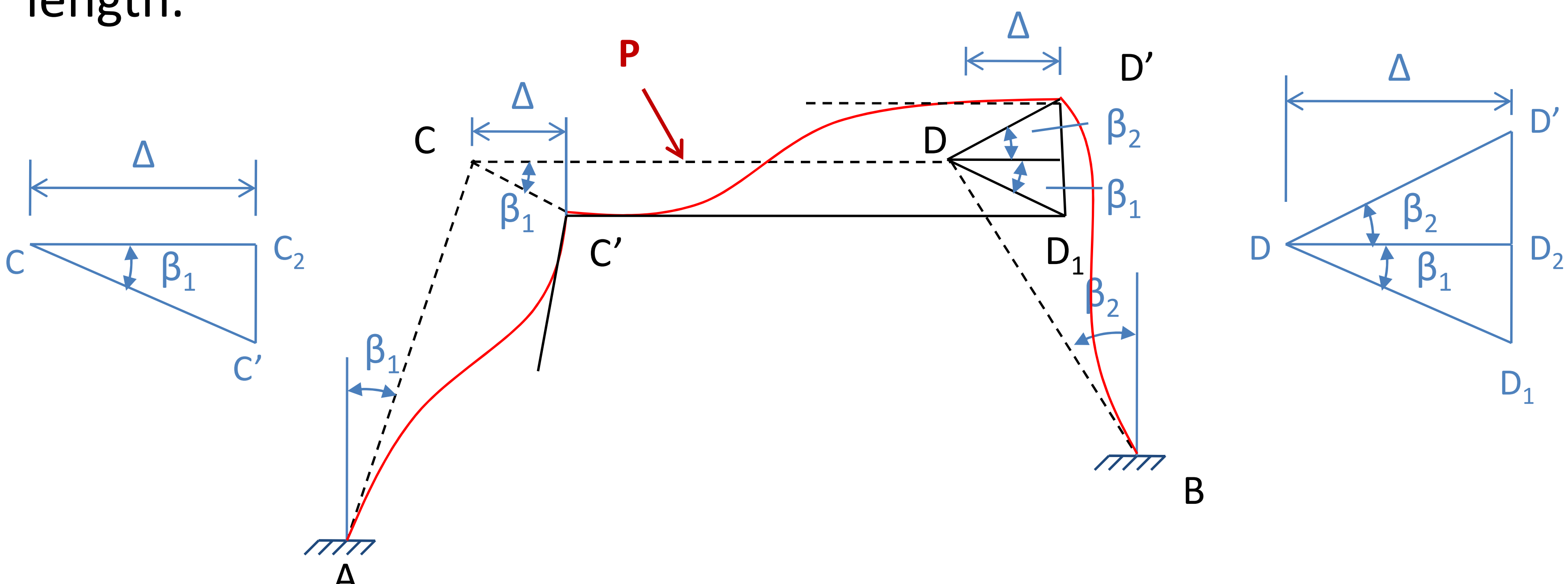
- Similarly, since member BD is also assumed to be inextensible, its end D can be moved only in the direction perpendicular to the member.
- Therefore, to obtain the deformed position of the joint D, we move the end D of member CD from its deformed position D_1 in the vertical direction and the end D of member BD in a direction perpendicular to BD, until the two ends meet at point D', where they are reconnected to obtain the displaced position D' of joint D.



Deflected shape of the frame due to sidesway

- By assuming that joint D does not rotate, we draw the elastic curves C'D' and BD', respectively, of member CD and BD, to complete the deflected shape of the entire frame.

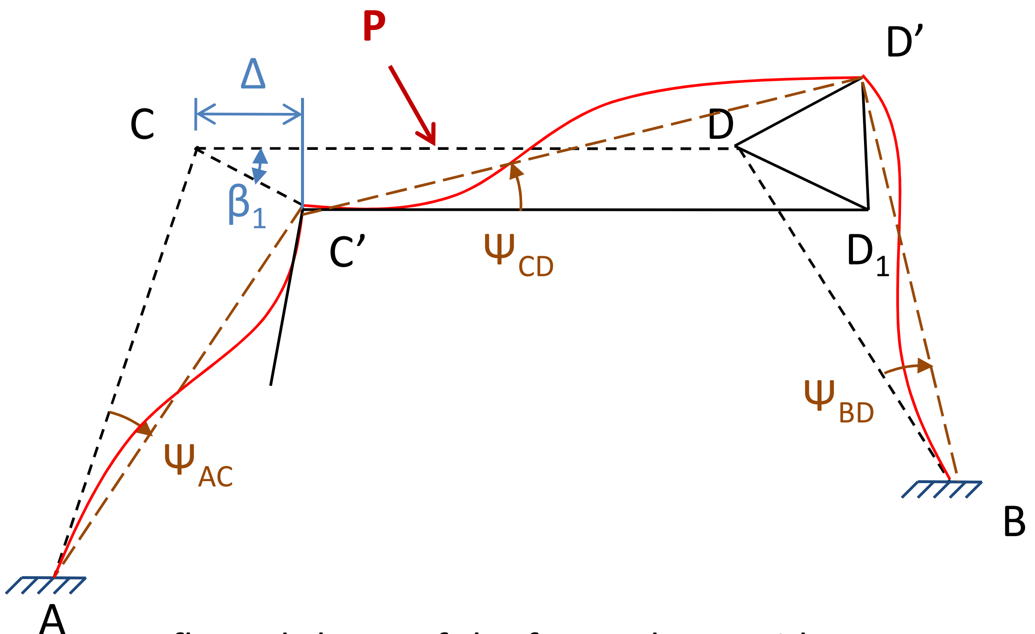
- The chord rotation of a member can be can be obtained by dividing the relative displacement between the two ends of the member in the direction perpendicular to the member, by the member's length.



Deflected shape of the frame due to sidesway

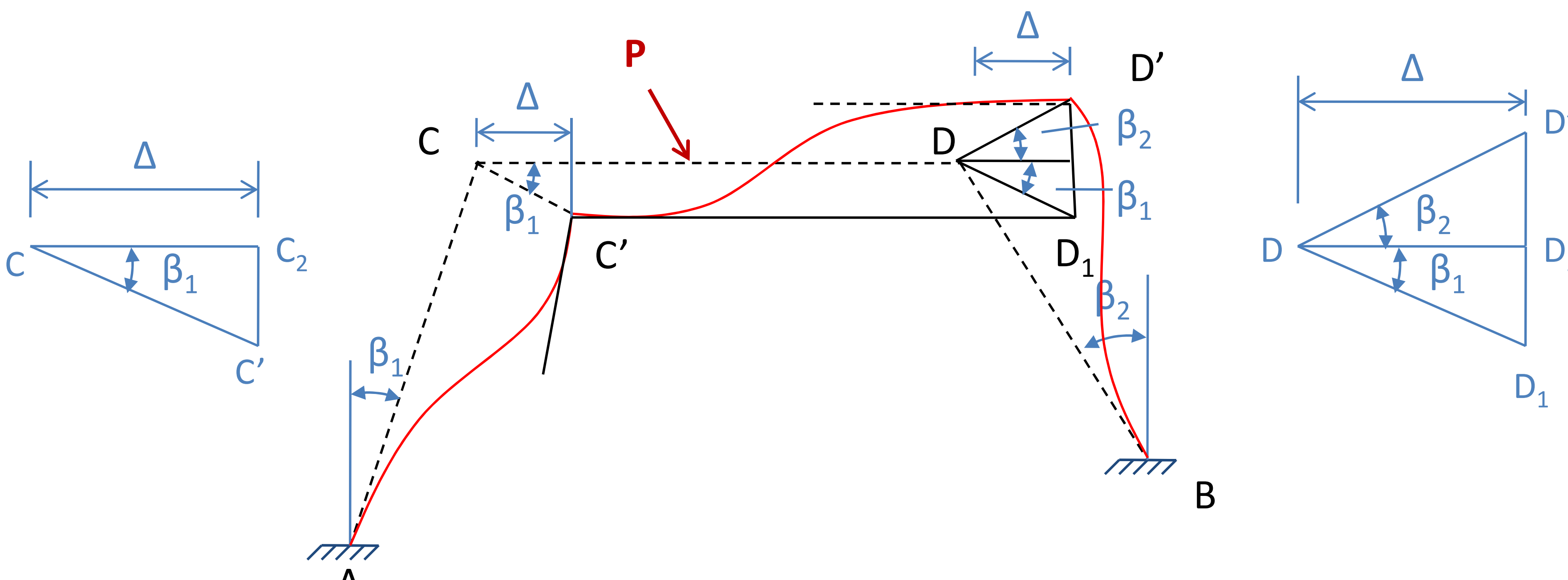
- Chord rotations of the three members of the frame are given by

$$\psi_{AC} = -\frac{CC'}{L_1} \qquad \psi_{BD} = -\frac{DD'}{L_2} \qquad \psi_{CD} = -\frac{D_1D'}{L} \tag{26}$$



Deflected shape of the frame due to sidesway

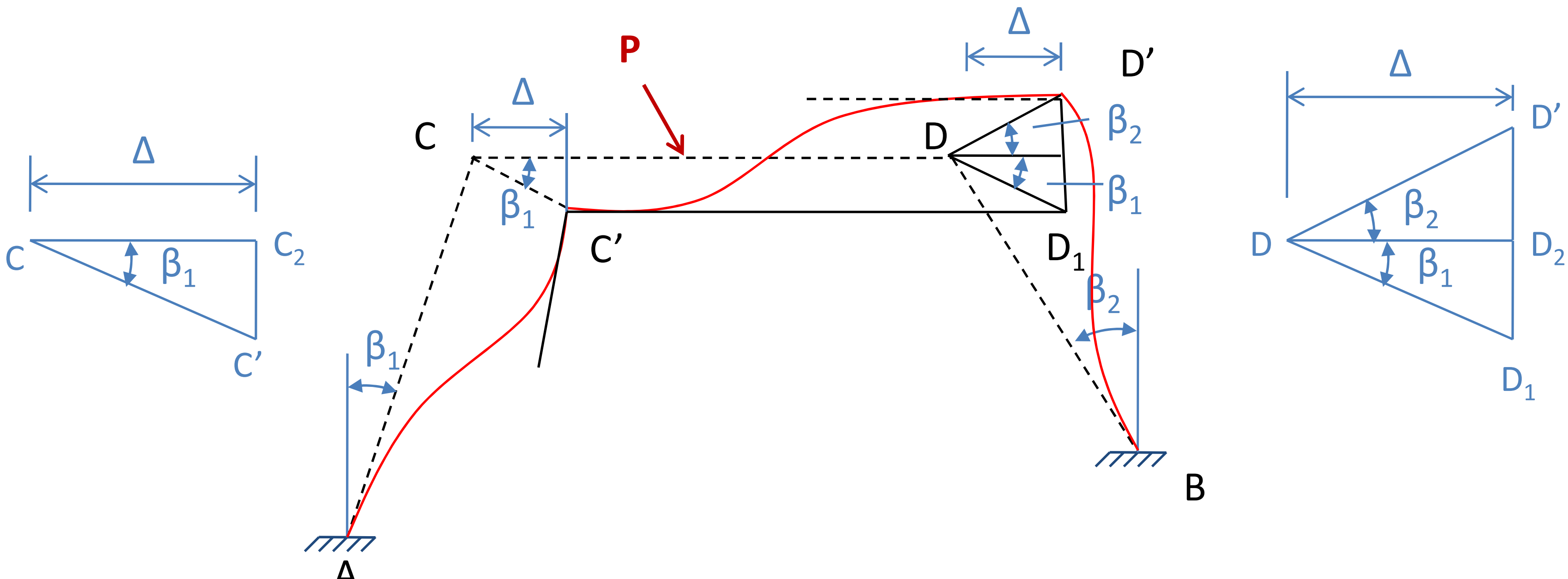
- The three chord rotations can be expressed in terms of the joint displacements Δ by considering the displacement diagrams of joints C and D as shown.
- Since CC’ is perpendicular to AC, which is inclined at an angle β_1 with the vertical, CC’ must make the same angle β_1 with the horizontal.



Deflected shape of the frame due to sidesway

- Thus, from the displacement diagram of joint C (triangle $CC'C_2$), we can see that

$$CC' = \frac{\Delta}{\cos \beta_1} \tag{27}$$

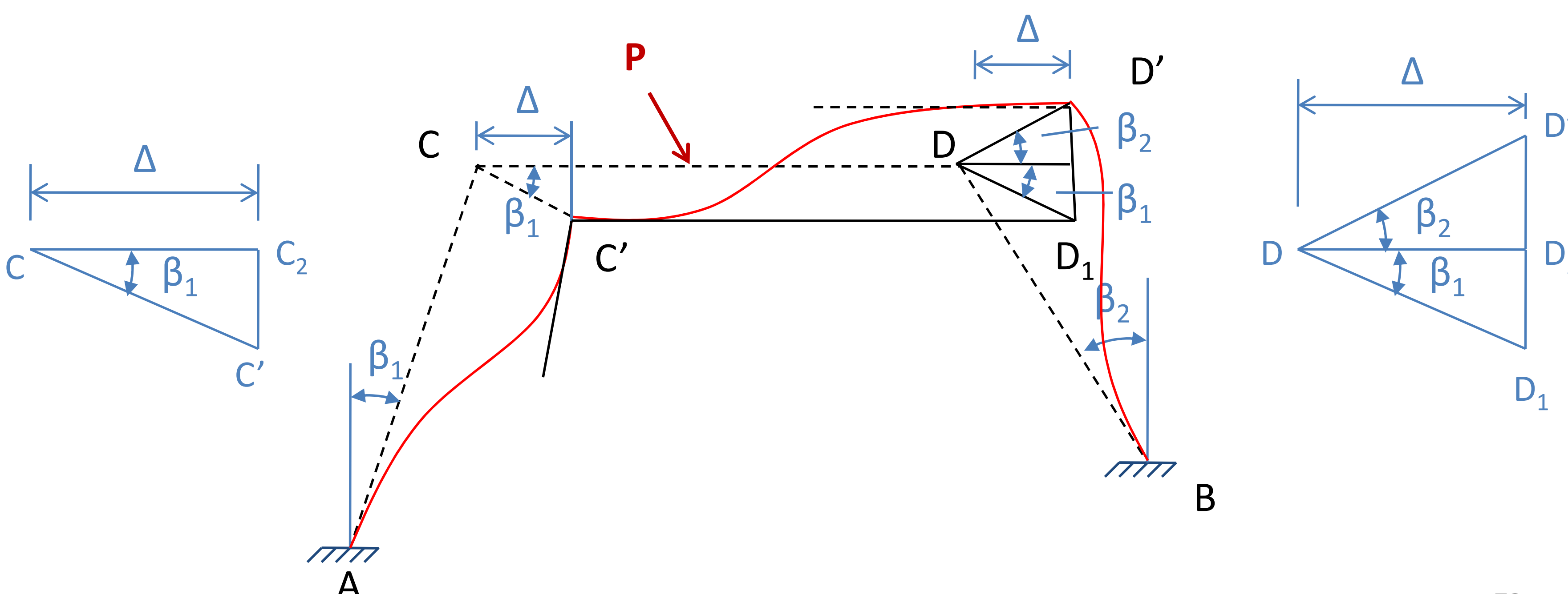


Deflected shape of the frame due to sidesway

- Next, let us consider the displacement diagram of joint D (triangle DD_1D'). It has been shown that DD_1 is equal in magnitude and parallel to CC'. Therefore,

$$DD_2 = DD_1 \cos\beta_1 = \Delta$$

- Since DD' is perpendicular to member BD, it makes an angle β_2 with the horizontal.



Deflected shape of the frame due to sidesway

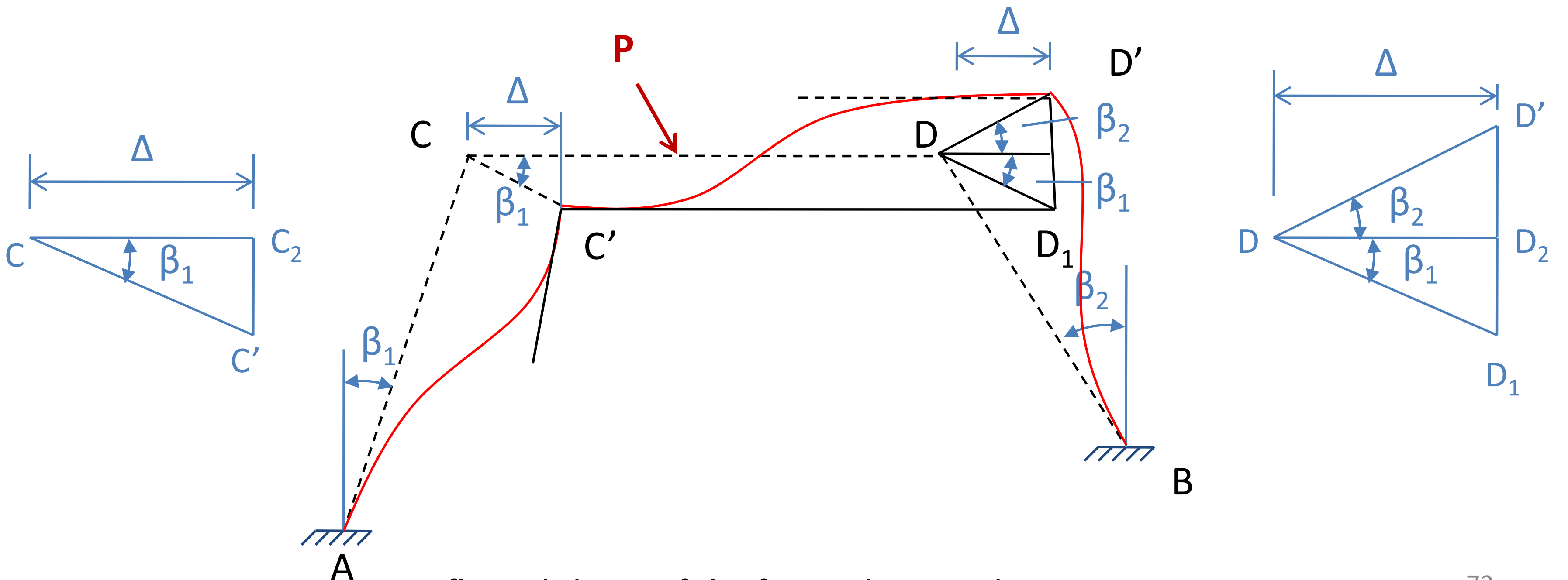
- From the displacement diagram of joint D,

$$DD' = \frac{DD_2}{\cos\beta_2} = \frac{\Delta}{\cos\beta_2} \tag{28}$$

and

$$D_1D' = DD_1 \sin\beta_1 + DD' \sin\beta_2 = \frac{\Delta}{\cos\beta_1}\sin\beta_1 + \frac{\Delta}{\cos\beta_2} sin\beta_2$$

$$D_1D' = \Delta(\tan\beta_1 + \tan\beta_2) \tag{29}$$



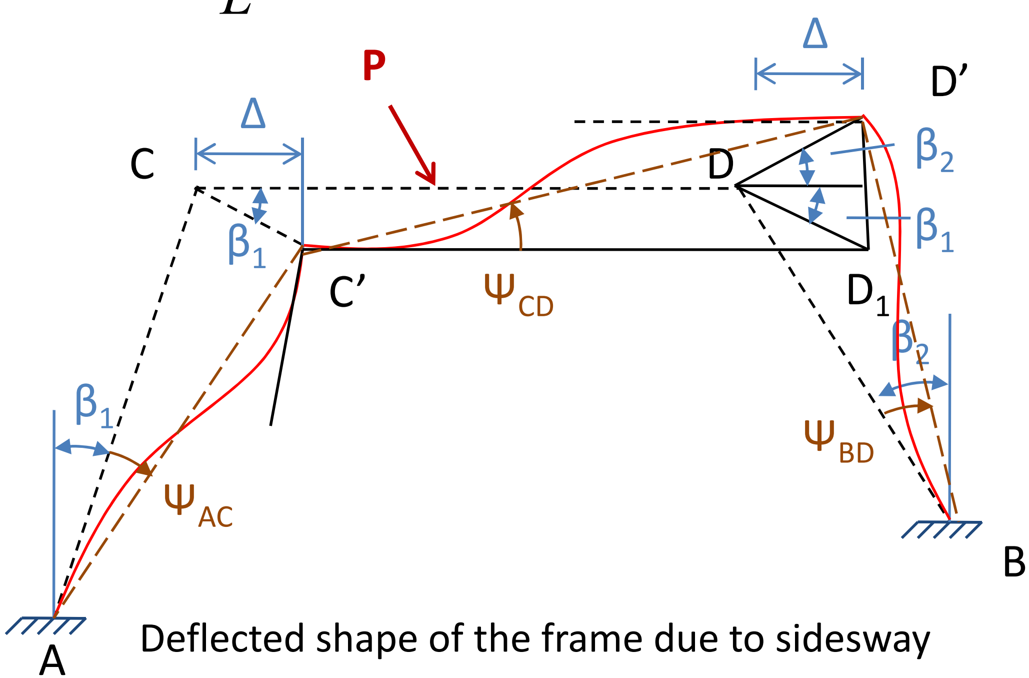
Deflected shape of the frame due to sidesway

- By substituting Eqs. 27 to 29 into Eq. 26, we obtain the chord rotations of the three members in terms of Δ.

$$\psi_{AC} = -\frac{\Delta}{L_1 \cos \beta_1} \tag{30a}$$

$$\psi_{BD} = -\frac{\Delta}{L_2 \cos \beta_2} \tag{30b}$$

$$\psi_{CD} = \frac{\Delta}{L}\left(\tan\beta_1 + \tan\beta_2\right) \tag{30c}$$



Deflected shape of the frame due to sidesway

- These expressions can be used to write the slope deflection equations, thereby relating member end moments to the three unknown joint displacements, θ_C, θ_D, and Δ.

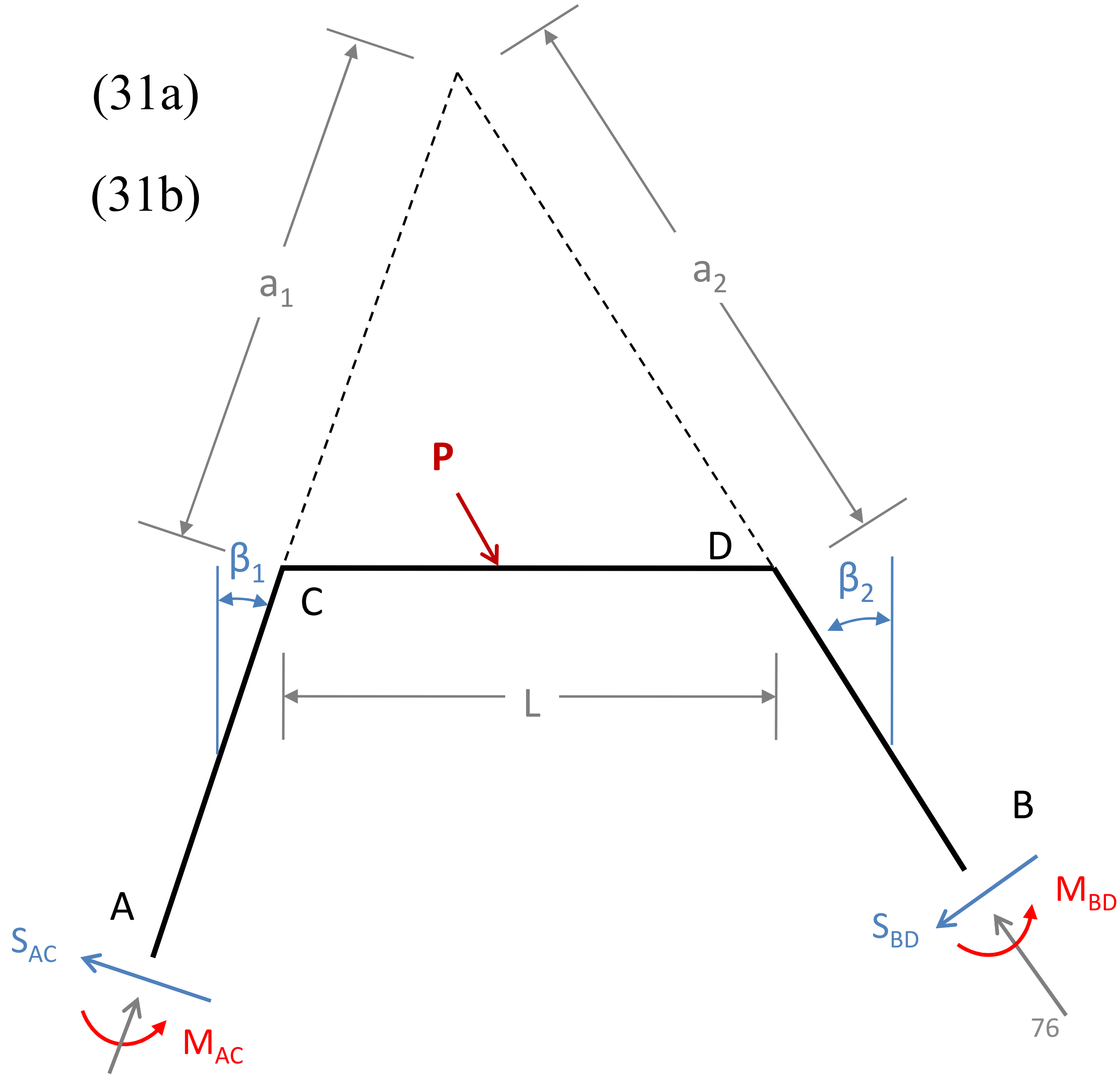
- As in case of rectangular frames, the three equilibrium equations necessary for the solution of unknown joint displacements can be established by summing the moments acting on joints C and D and by summing the horizontal forces acting on the entire frame.

- For frames with inclined legs, it is usually more convenient to establish the third equilibrium equation by summing the moments of all the forces and couples acting on the entire frame about a moment centre O, which is located at the intersection of the longitudinal axes of the two inclined members, as shown in the next slide.

- The location of the moment centre O can be determined by using the conditions

$$a_1 \cos \beta_1 = a_2 \cos \beta_2 \qquad (31a)$$

$$a_1 \sin \beta_1 + a_2 \sin \beta_2 = L \qquad (31b)$$



- By solving Eqs. (31a & 31b) simultaneously for a_1 and a_2, we obtain

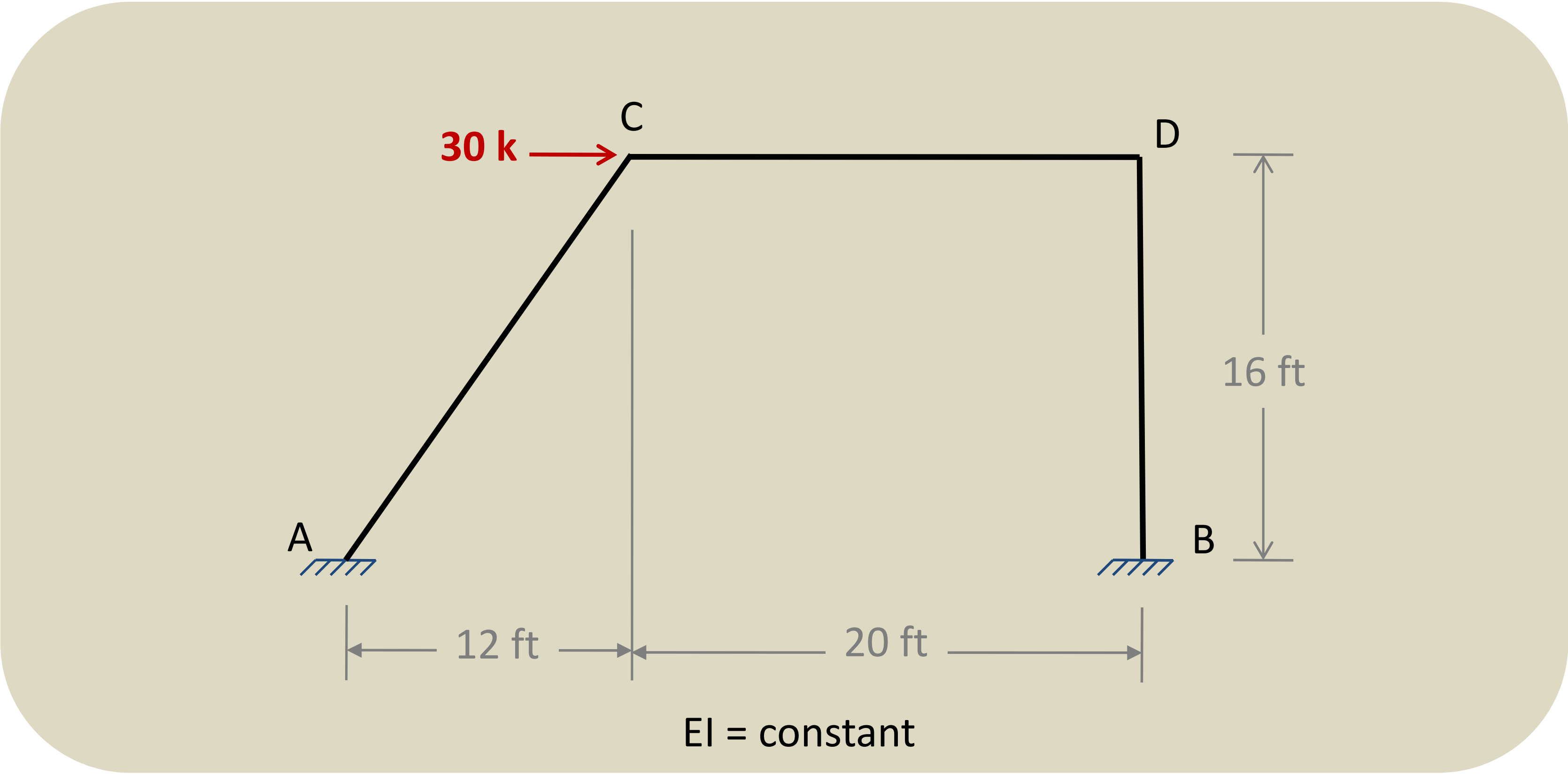
$$a_1 = \frac{L}{\cos\beta_1(\tan\beta_1 + \tan\beta_2)} \qquad (32a)$$

$$a_2 = \frac{L}{\cos\beta_2(\tan\beta_1 + \tan\beta_2)} \qquad (32b)$$

- Once the equilibrium equations have been established, the analysis can be completed in the usual manner, as discussed previously.

Example 4

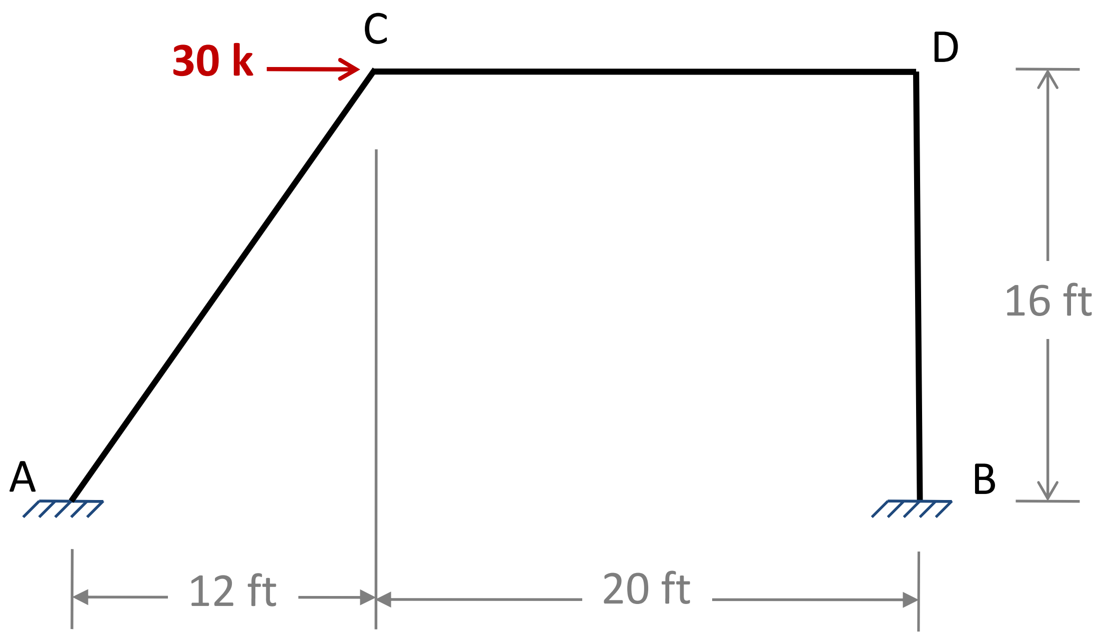
- Determine the member end moments and reactions for the frame shown by the slope-deflection method.



Solution

1. Degree of Freedom

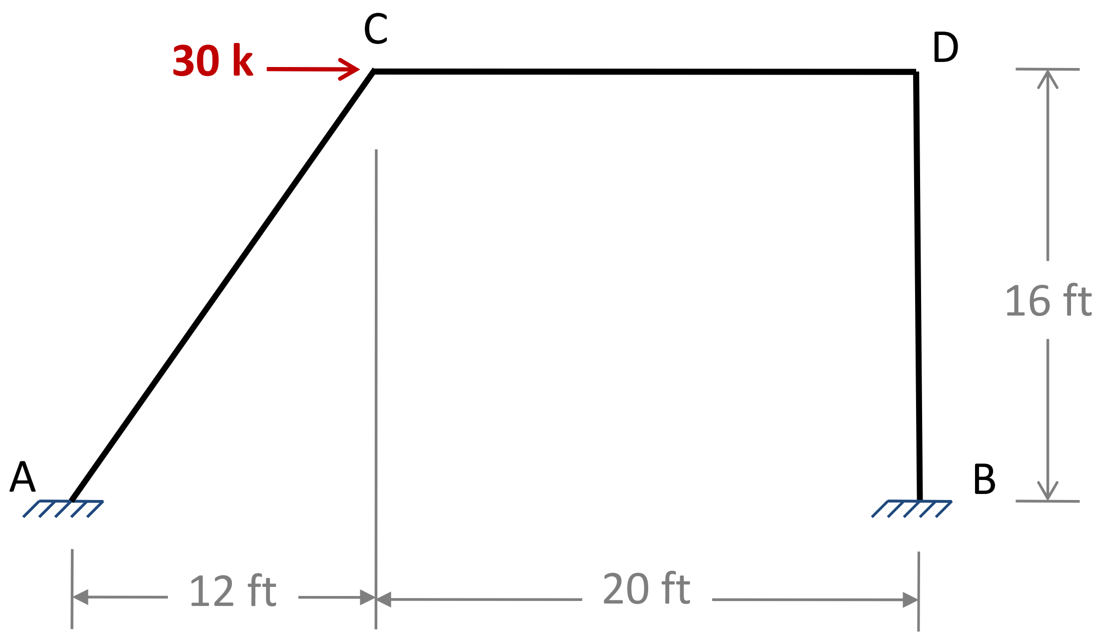
$$\theta_C, \theta_D \text{ and } \Delta$$



EI = constant

2. Fixed End Moments

Since no external loads are applied to the members, the fixed-end moments are zero.

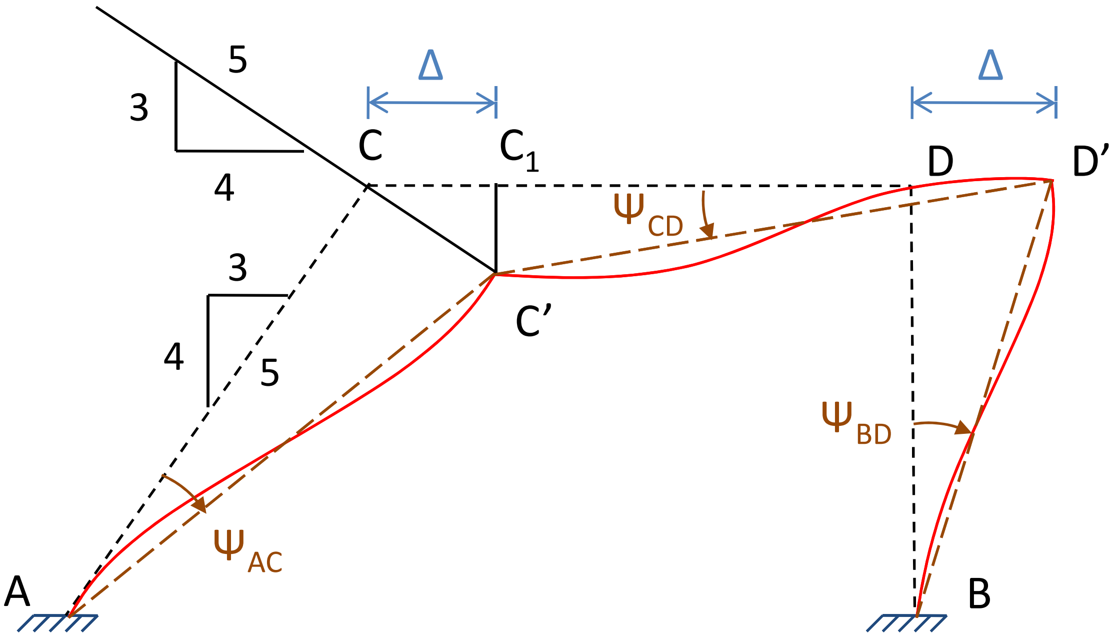


EI = constant

3. Chord Rotations

$$\psi_{AC} = -\frac{CC'}{20} = -\frac{\left(\frac{5}{4}\right)\Delta}{20} = -0.0625\Delta \qquad \psi_{BD} = -\frac{DD'}{16} = -\frac{\Delta}{16} = -0.0625\Delta$$

$$\psi_{CD} = \frac{C'C_1}{20} = \frac{\left(\frac{3}{4}\right)\Delta}{20} = 0.0375\Delta$$



4. Slope-Deflection Equations

$$M_{AC} = \frac{2EI}{20}\left(\theta_C - 3\left(-0.0625\Delta\right)\right) = 0.1EI\theta_C + 0.0188EI\Delta \qquad (1)$$

$$M_{CA} = \frac{2EI}{20}\left(2\theta_C - 3\left(-0.0625\Delta\right)\right) = 0.2EI\theta_C + 0.0188EI\Delta \qquad (2)$$

$$M_{BD} = \frac{2EI}{16}\left(\theta_D - 3\left(-0.0625\Delta\right)\right) = 0.125EI\theta_D + 0.0234EI\Delta \qquad (3)$$

$$M_{DB} = \frac{2EI}{16}\left(2\theta_D - 3\left(-0.0625\Delta\right)\right) = 0.25EI\theta_D + 0.0234EI\Delta \qquad (4)$$

$$M_{CD} = \frac{2EI}{20}\left(2\theta_C + \theta_D - 3\left(0.0375\Delta\right)\right) = 0.2EI\theta_C + 0.1EI\theta_D - 0.0113EI\Delta \qquad (5)$$

$$M_{DC} = \frac{2EI}{20}\left(2\theta_D + \theta_C - 3\left(0.0375\Delta\right)\right) = 0.2EI\theta_D + 0.1EI\theta_C - 0.0113EI\Delta \qquad (6)$$

5. Equilibrium Equations

By considering the moment equilibrium of joints C and D, we obtain the equilibrium equations

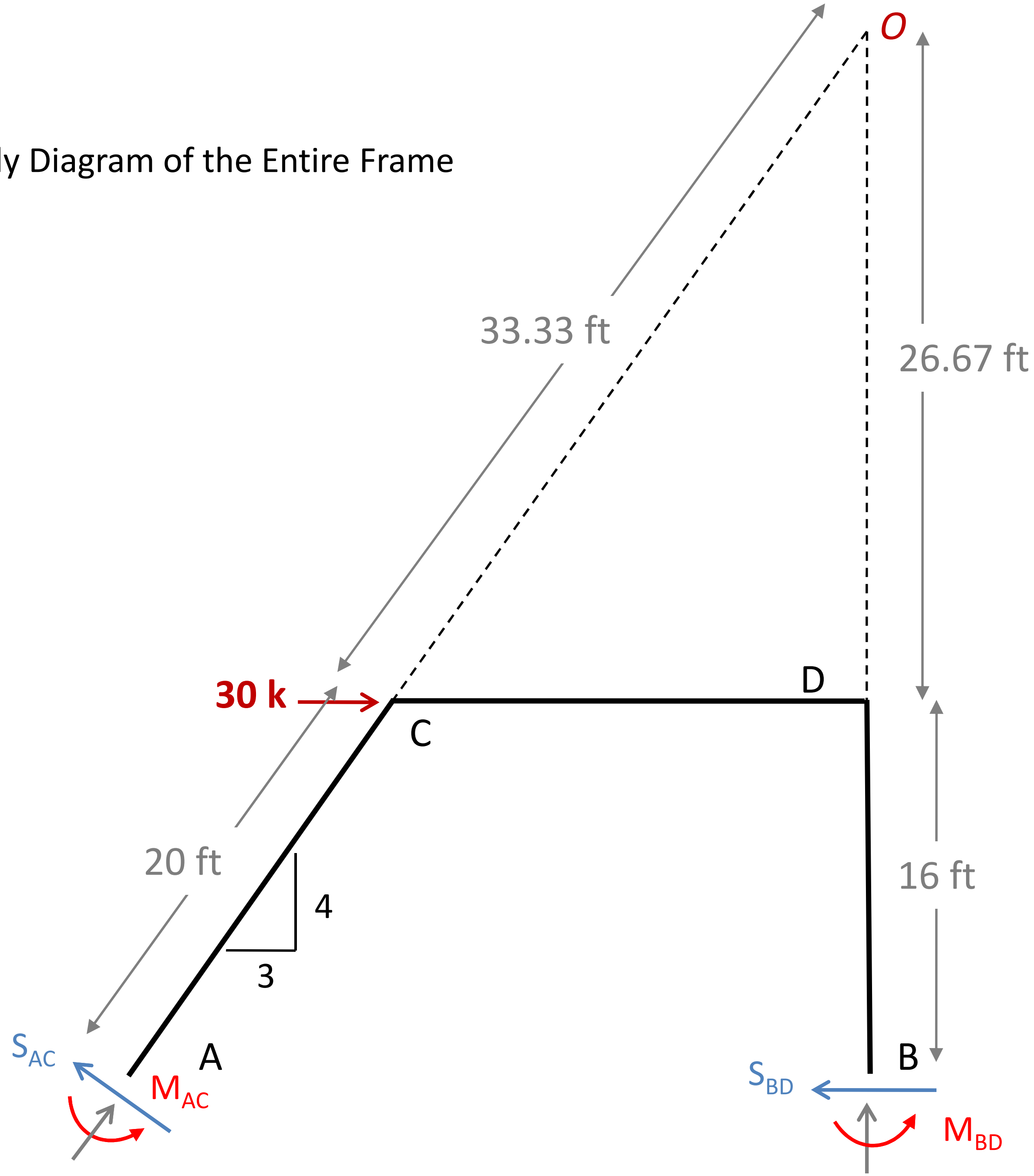
$$M_{CA} + M_{CD} = 0 \tag{7}$$

$$M_{DB} + M_{DC} = 0 \tag{8}$$

The third equilibrium equation is established by summing the moments of all the forces and couples acting on the free body of the entire frame about point *O*, which is located at the intersection of the longitudinal axes of the two columns as shown.

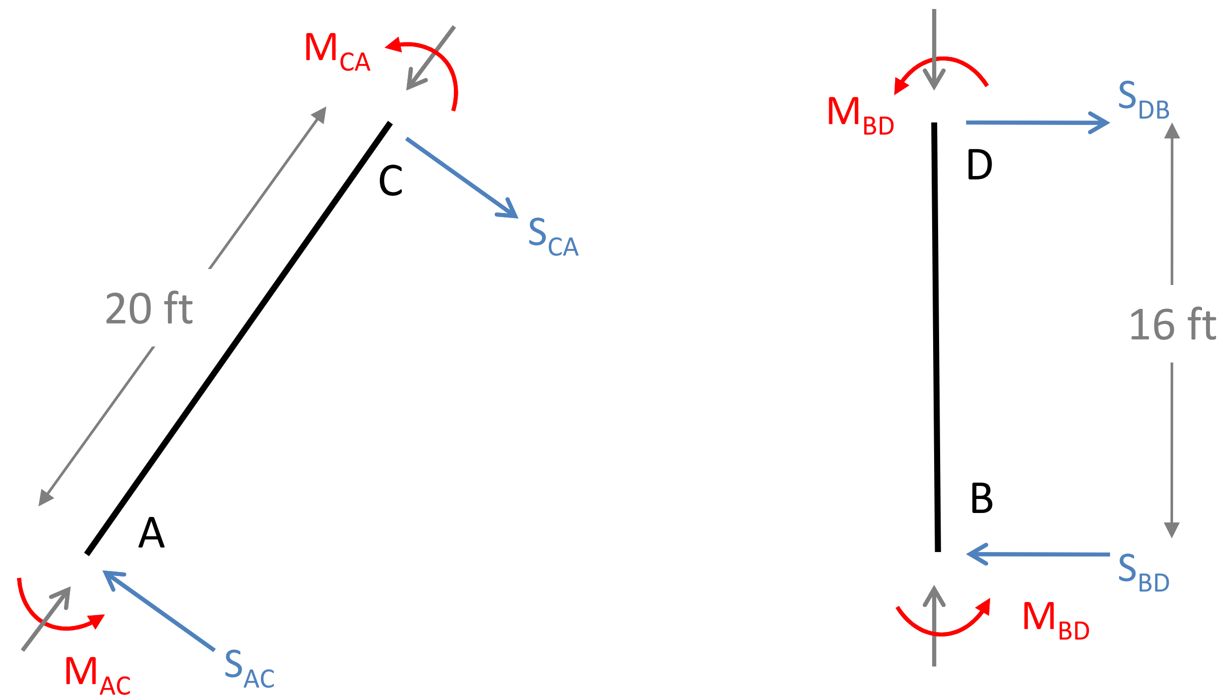
$$+\circlearrowleft \sum M_O = 0 \qquad M_{AC} - S_{AC}(53.33) + M_{BD} - S_{BD}(42.67) + 30(26.67) = 0$$

Free-body Diagram of the Entire Frame



$$+\circlearrowleft \sum M_O = 0 \qquad M_{AC} - S_{AC}(53.33) + M_{BD} - S_{BD}(42.67) + 30(26.67) = 0$$

in which the shears at the lower ends of the columns can be expressed in terms of column end moments as shown below.



$$S_{AC} = \frac{M_{AC} + M_{CA}}{20} \qquad S_{BD} = \frac{M_{BD} + M_{DB}}{16}$$

by substituting these expressions into the third equilibrium equation, we obtain

$$1.67M_{AC} + 2.67M_{CA} + 1.67M_{BD} + 2.67M_{DB} = 800 \qquad (9)$$

6. Joint Displacements

Substitution of the slope-deflection equations Eqs. 1 to 6 into the equilibrium equations Eqs. 7 to 9 yields

$$0.4EI\theta_C + 0.1EI\theta_D + 0.0075EI\Delta = 0 \qquad (10)$$

$$0.1EI\theta_C + 0.45EI\theta_D + 0.0121EI\Delta = 0 \qquad (11)$$

$$0.71EI\theta_C + 0.877EI\theta_D + 0.183EI\Delta = 800 \qquad (12)$$

By solving Eqs. 10 through 12 simultaneously, we determine

$$EI\theta_C = -66.648 \text{ k-ft}^2$$

$$EI\theta_D = -125.912 \text{ k-ft}^2$$

$$EI\Delta = 5{,}233.6 \text{ k-ft}^3$$

7. Member End Moments

By substituting the numerical values of $EI\theta_C$, $EI\theta_D$, and $EI\Delta$ into the slope-deflection equations (Eqs. 1 through 6), we obtain

$$M_{AC} = 91.7\ \text{k-ft} \qquad \text{ANS}$$

$$M_{CA} = 85.1\ \text{k-ft} \qquad \text{ANS}$$

$$M_{BD} = 106.7\ \text{k-ft} \qquad \text{ANS}$$

$$M_{DB} = 91\ \text{k-ft} \qquad \text{ANS}$$

$$M_{CD} = -85.1\ \text{k-ft} \quad \text{or} \quad 85.1\ \text{k-ft} \qquad \text{ANS}$$

$$M_{DC} = -91\ \text{k-ft} \quad \text{or} \quad 91\ \text{k-ft} \qquad \text{ANS}$$

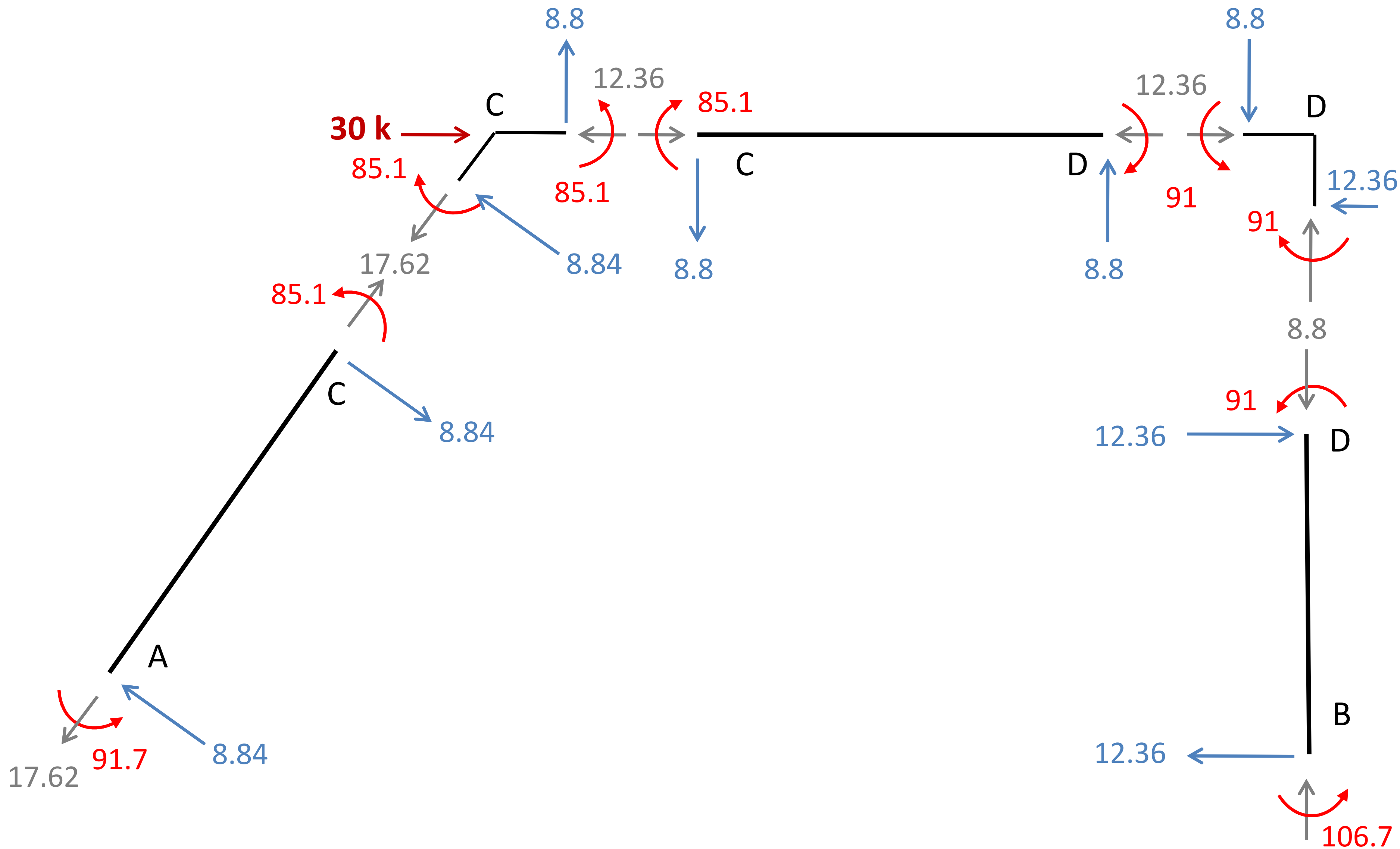
Back substitution of the numerical values of member end moments into the equilibrium equations yields

$$M_{CA} + M_{CD} = 85.1 - 85.1 = 0 \qquad \text{Checks}$$

$$M_{DB} + M_{DC} = 91 - 91 = 0 \qquad \text{Checks}$$

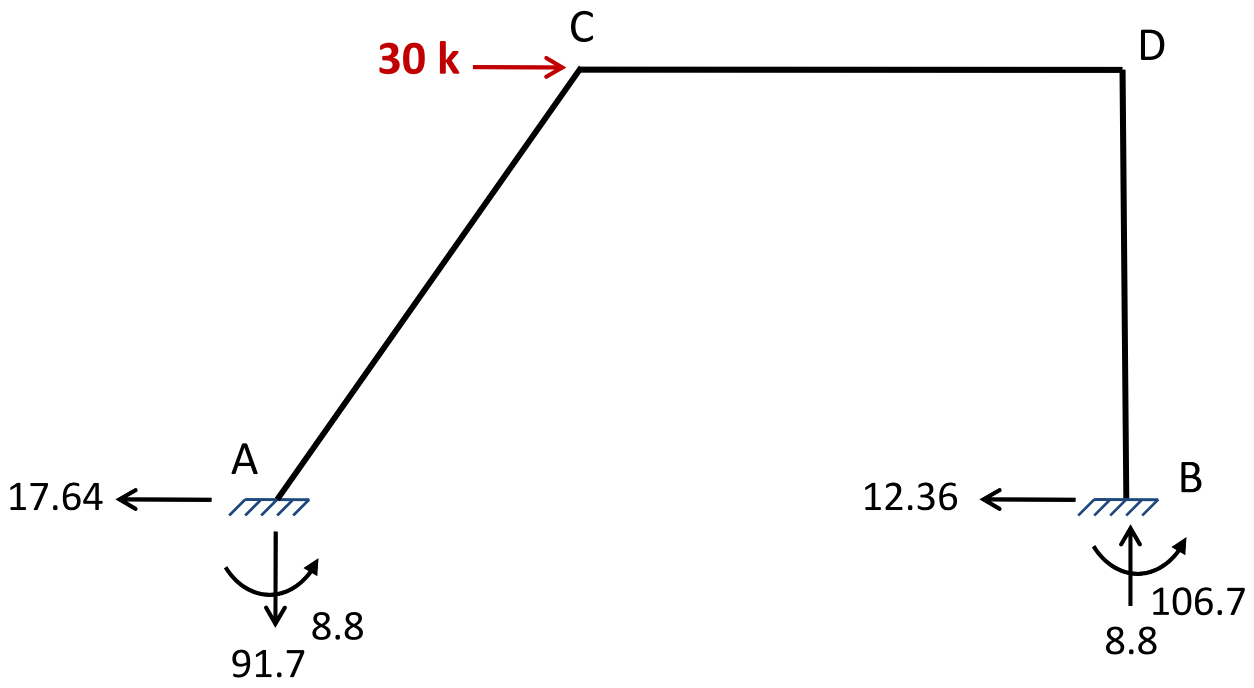
$$\begin{aligned} 1.67M_{AC} + 2.67M_{CA} + 1.67M_{BD} + 2.67M_{DB} &= 1.67(91.7) + 2.67(85.1) \\ &\quad + 1.67(106.7) + 2.67(91) \\ &= 801.5 \cong 800 \qquad \text{Checks} \end{aligned}$$

8. Member End Shears and Axial Forces



Member End Moments, Shears and Axial Forces

9. Support Reactions



Member End Moments, Shears and Axial Forces

Thank You